ALGEBRA

CLASS V

PREPARED BY

NCERT
MATHEMATICS STUDY GROUP
UNIVERSITY OF RAJASTHAN, JAIPUR

NATIONAL COUNCIL OF EDUCATIONAL
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PREFACE

For the last ten or twelve years a revolution, popularly known as the 'New Mathematics Revolution' has swept the world. It has involved an expenditure of millions of dollars, roubles, pounds, francs and rupees. What is more important is that it has involved millions of man-hours of school teachers, college teachers, educational administrators and some of the foremost mathematicians of modern times. It has changed the attitude to mathematics of billions of childern, teachers and parents.

About a hundred projects* have been set up all over the world for developing new curricular materials including text books, teacher' guides, charts, models, film strips, films, T.V. lessons, etc and for training teachers in new ideas. In India, the efforts for improving school mathematics have included setting up a panel of distinguished mathematicians under the chairmanship of Prof. Ram Bihari, an editorial board under my chief editorship, a project under the auspices of the Department of Science Education of the NCERT, organisation of six study groups at Banglore, Baroda, Delhi, Jaipur, Jadavpur and Kanpur, a large number of summer institutes for training of teachers and a national conference on school mathematics.†

The six study groups for developing new curricular materials in mathematics were set up in 1966 as a result of the decisions of the Conference on Mathematics and Science Education held under the chairmanship of Prof. D. S. Kothari. The coordinating committee of the Mathematics Study Groups decided to assign responsibilities as follows :

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* Lock D Report of the International Clearing House on Curricular Development in Science Education, Maryland University, U S.A

† Kapur, J N. : Proceedings of the National Conference on School Mathematics, Mathematical Association of India

Primary Mathematics (class I-IV) and Applications of Mathematics to Kanpur Study Group, Algebra (including Arithmetic) for classes V to VII and VIII to X to Baroda, Delhi and Jaipur Groups and Geometry (including co-ordinate Geometry and Trigonometry) for classes V to VII and VIII to X to Banglore and Jadavpur Groups. The entire work was to be co-ordinated by the Co-ordinating Committee consisting of the Directors of the Study Groups.

So far the Study Groups have prepared the following books :

(i) Hand books for Teachers of classes I and II,
(ii) Algebra text-books and their Teachers' Guides for classes V, VI and VII,
(iii) Geometry text-books and their Teachers' Guides for classes V, VI and VII.

Books for secondary level are under preparation.

Experimental editions of these books are being made available for restricted use now for try-outs in some selected schools. Their final editions shall be available for open use by State Governments later, after necessary improvements have been made in the light of the reactions of teachers and pupils. The final version of the Algebra books has been prepared by the Jaipur Group. For sometime Shri P. K. Srinivasan of the Kanpur Study Group was also attached with the Jaipur Study Group. The Teachers' Guides of Algebra books have been prepared by the Jadavpur Group.

The final version of the Geometry text-books has been prepared by the Banglore Group and that of the Geometry Teachers' Guides by the Jadavpur Group.

In Algebra, the approach has been through sets, truth sets of open sentences including inequalities and structures of the systems of natural numbers, integers and rational numbers. In Geometry, the approach has been through transformations and symmetries. Both the approaches are likely to be new for teachers and therefore all efforts will be made to train the teachers in these ideas. Even when training programmes are not available, it is hoped that the teachers' handbooks will be sufficiently lucid and clear to provide sufficient help to interested teachers.

Most of the ideas contained in these books have been tried out with many batches of students and the reaction of students and teachers has been of unmitigated enthusiasm. These were also discussed at the National Workshop on School Mathematics held at I.I.T. Kanpur in December 1968 and attended by representatives of eleven State Institutes of Education Their enthusiastic support has been a source of strength to us.

In the present Algebra text, the answers to questions, which are oral, marked with asterisk (*) or have many answers, have not been generally given.

I take this opportunity of thanking Prof. D.S. Kothari, Chairman of the Co-ordinating Committee of all Science and Mathematics Study Groups, Dr. S. V. C. Aiya, Director, NCERT, Shri Rajender Prasad, former Field Officer NCERT, Sarve Shri R.C. Sharma, G.S. Baderia, and K.V. Rao, Readers in Mathematics, NCERT for administrative support at all levels without which production and printing of these books would not have been possible.

On the academic side, I would like to thank all the directors and members of the Study Groups whose names are given elsewhere for their co-operation. I must make special mention of Prof. K. Venkatachaliengar whose group prepared the first detailed curriculum of Geometry and the final version of the Geometry books, of Prof. G. C. Patni for giving valuable leadership to the Jaipur Group during the preparation of Algebra books and of Dr. D. K. Sinha for getting Geometry and Algebra Teachers' Guides prepared.

I am thankful to Shri P.C. Bhargava, Manager, Rajasthan University Press, Jaipur for his sincere co-operation in bringing out the Algebra Text-books.

J.N. Kapur
Convener,
NCERT Mathematics Study Groups.

December 25, 1970.

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CONTENTS

Chapter 1 : SETS—AN INTRODUCTION 1-25
1—1 Sets and Elements 1
1—2 Sets and Symbols 6
1—3 Venn Diagrams 8
1—4 Equal Sets 9
1—5 Set and its Subsets 12
1—6 Intersection of Sets 18
1—7 Union of Sets 20

Chapter 2 : THE SET OF WHOLE NUMBERS : W 26-121
2—1 Comparison of Sets 26
2—2 The Whole Numbers 31
2—3 Decimal Numerals 36
2—4 Number-Ray and Order Relation 42
2—5 Addition in W 46
2—6 Properties through Patterns 50
2—7 Addition Process 56
2—8 Subtraction in W 57
2—9 Subtraction Procedure involving Large Numbers 63
2-10 Application of Subtraction 64
2-11 Multiplication in W 66
2-12 Properties of Multiplication through Patterns 76
2-13 Multiplication Procedure for large numbers 85
2-14 Division in W 88
2-15 Applications of Division 95
2-16 Exponents 97
2-17 Factors and Multiples 100
2-18 Even and Odd Numbers 104
2-19 Prime and Composite Numbers 108
2-20 L.C.M. and H.C.F. of Natural Numbers 115

Chapter 3 : THE SET OF FRACTIONAL NUMBERS 122-205
3—1 Introduction 122
3—2 Equivalent fractions 128
3—3 Order 136
3—4 Addition in Q_+ 144
3—5 Properties of Addition in Q_+ 151

3—6 Subtraction in Q_+ 155
3—7 Multiplication in $Q+$ 161
3—8 Properties of Multiplication in $Q+$ 164
3—9 Exponents 172
3-10 Division in $Q+$ 174
3-11 Properties of Division in $Q+$ 178
3-12 Fractions as Ratios 180
3-13 Fractional Numbers in other forms 184
3-14 Decimal Fractions (Addition and Subtraction) 192
3-15 Decimal Fractions (Multiplication and Division) 196
3-16 Some Applications of decimal fractions 202

Chapter 4 : SENTENCES 206 219
4—1 Sentences 206
4—2 Open Sentences 206
4—3 Solution Sets of Open Sentences 207
4—4 Graphs of Solution Sets of Open Sentences 208
4—5 Sentences With Other Connectives 209
4—6 Implications 211
4—7 Equations and Solution Sets 214

Chapter 5 : APPLICATIONS 220 246
5—1 Introduction 220
5—2 Interest and Principal 226
5—3 Interest and Time 232
5—4 Interest and Rate 234
5—5 Quantity and Cost 238
5—6 Time and Work 240
5—7 Work and Number of Men 242
5—8 Time and Distance 244
Answers to Exercise Sets 247-270

1

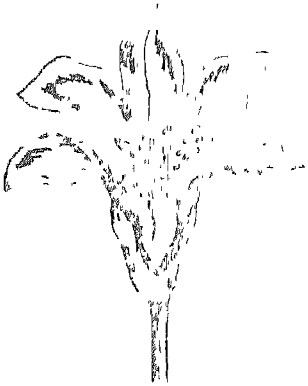
SETS—AN INTRODUCTION

1-1 Sets and Elements

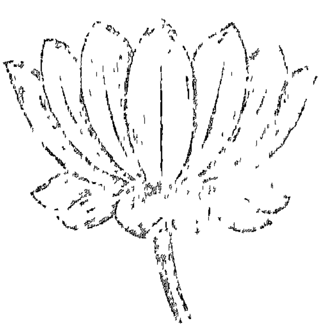
In our daily life we often talk of a collection of objects. For example, consider the class in which you are reading. Shown a person, you can say if he or she belongs to the class or not. Again, consider all those students who are selected to play a cricket match. All these players belong to the team, others do not belong to the team Similarly, think of the members of your family. Your parents and you belong to your family, while your classmates may or may not belong to your family. You must have observed that we are considering a class, a team, or a family. A class is a *collection* of students. A team is a *collection* of players. A family is a *collection* of related persons.

In mathematics we often use the word **set** for a collection and we are interested in set of numbers, set of points, set of geometrical figures and so on; for example, the set of even numbers, the set of integers, the set of points in a circle, the set of triangles etc. You will learn, later on, that the concept of a set is of frequent use in mathematics and other sciences, too.

ROSE



LILY



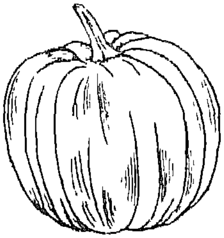
LOTUS

Look at the set of objects given on page 1.

How can you describe it ?

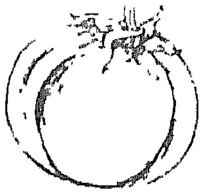
It is a set of flowers.

The figure given below is a set of vegetables.



PUMPKIN

CABBAGE



TOMATO

In a set of flowers, we say that flowers are *members* or *elements* of the set. For example, in the set of flowers on page 1, rose, lily and lotus are the elements of the set. Do you know the elements in the set of vegetables given above ?

They are pumpkin, cabbage and tomato.

Let us consider the set of even numbers less than 10. You can easily say what its elements are. They are 0, 2, 4, 6, 8. Any even number less than 10 belongs to it and no other number can belong to it.

Similarly, if you consider the set of odd numbers less than 7, the elements of this set are 1, 3 and 5; and if you consider the set of letters needed to spell the word ORAL, its elements are O, R, A and L.

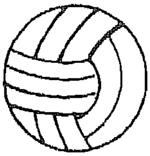
A *set* is said to be *well-defined,* if given an object, we can decide whether it is an element of the set or not. We always take the sets which we consider as well-defined sets.

Oral

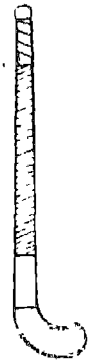
1. Do the following represent sets and if so, what are the elements in each set ?

(i)

BAT

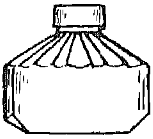


BALL

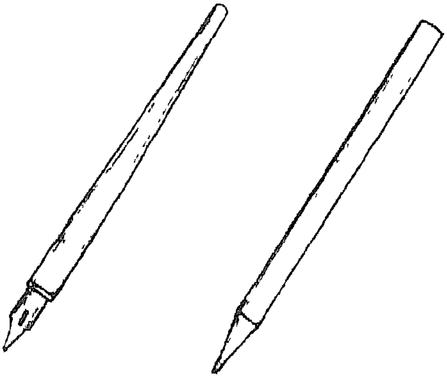


HOCKEY STICK

(ii)

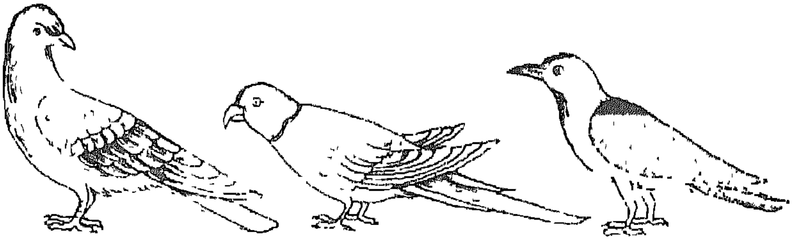


INKPOT



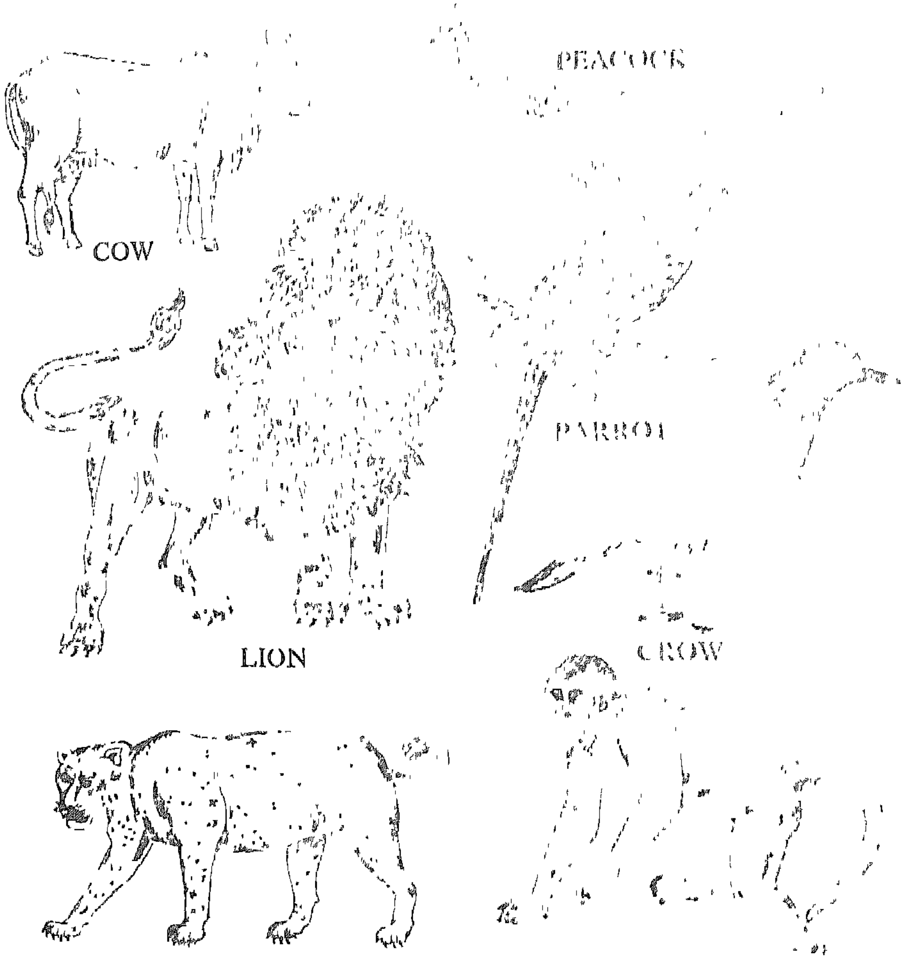
PEN PENCIL

(iii)



PIGEON PARROT CROW

2. Classify the following into two sets according to the number of legs of each animal. Give the description of each set.



3. State whether each of the following belongs to the set of fruits ?
   (a) cauliflower (b) banana (c) apple (d) rose (e) cabbage.

4. Answer the following :
   (a) Does August belong to the set of months in the English Calender ?
   (b) Does Sunday belong to the set of working days of your school ?

5. State whether each of the following statements is true :
   (a) 81 belongs to the set of two-digit numbers ending in 1.
   (b) 15 belongs to the set of numbers ending in 5 and lying between 20 and 40.

Written

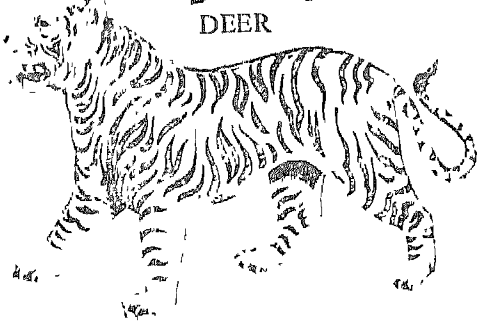
6. Classify the following into two sets having or not having horns. Give the description of each set.

ELEPHANT

BUFFALO

DEER

LION

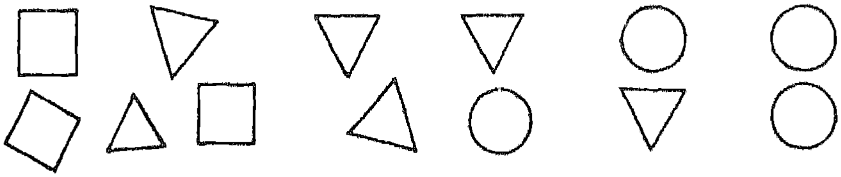


TIGER

BEAR

Written

7. Reclassify the following into sets, according to their shapes :



1-2 Sets and Symbols

Representation of a set : It is customary to list the elements of a set by writing them within a *pair of braces* (or *curly brackets*) as follows :

{ bat, ball, wicket }.

Similarly we can represent a set of writing material as

{ Note book, Pencil, Pen, Inkpot } ;

the set of numbers ending in 7 as

{ 7, 17, 27, 37, . . . } ;

the set of two-digit numbers as

{ 10, 11, 12, . . . , 98, 99 } .

This method of representing a set is known as '*Braces form.*' The *three dots* in the last two examples indicate that we have listed only a few of the elements. One can guess easily the other elements. The missing elements of the set are indicated by . . . .

Sometimes, we describe a set by giving a common characteristics of its members.

For example, the set

$$\{ 10, 11, 12, \ldots, 99 \}$$

is written as

{ two-digit numbers } .

This set of two-digit numbers is also written as

$$\{ x \mid x \text{ is a two-digit number} \}$$

and is read as, 'a set consisting of all x such that x is a two-digit number '

Similarly the set of numbers ending in 7 can be written as

$$\{ x \mid x \text{ is a number ending in } 7 \}$$

and is read as 'a set consisting of all x such that x is a number ending in 7'.

This form of expressing a set is known as *Set-Builder Form.*

Some of the sets considered are such that they contain a limited number of different elements so that while considering the elements of the set, the process of counting comes to an end. Such sets are called *finite sets*. On the other hand, if a set is not finite, it is called an *infinite set.* In an infinite set, we could never list all their elements. For example, in the above set of numbers ending in 7, we cannot write all the numbers ending in 7. This is a set consisting of an infinite number of elements just like the *set of points on a line* or *the set of all counting numbers*, etc.

It is usual to denote sets by *capital letters* A, B, C, etc., and elements, wherever necessary, by *lower-case letters* a, b, c, etc. Thus we can write :

$$\mathrm{A} = \{ b, i, t \},$$

$$\mathrm{B} = \{ a, e, i, o, u \},$$

$$\mathrm{C} = \{ 0, 10, 20 \},$$

$$D = \{ \text{two digit numbers} \},$$

$$K = \{ 0, 1, 2, \ldots, 9 \}.$$

Let E be the set of numbers ending in 8.

Does 128 belong to the set E ?

Yes, 128 is an element of E.

We express this fact symbolically as

$$128 \in E$$

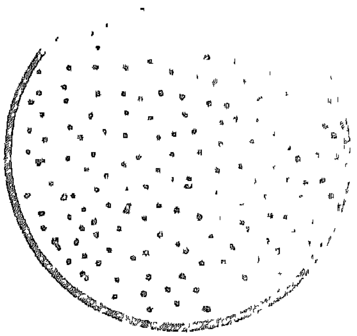
The symbol $\in$ is an abbreviated (short) form for '*is an element of*' or '*belongs to.*'

The symbol $\notin$ is a short form of '*is not a member of*' or '*does not belong to.*' Thus,

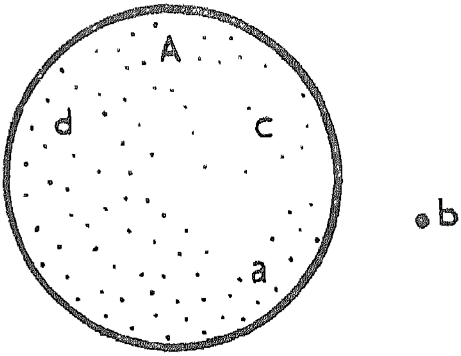
$$182 \notin E.$$

1-3 Venn Diagrams

We often draw diagrams to show what the elements of a set are ? Such diagrams help us to understand ideas in a better way. These diagrams are called *Venn Diagrams.* These diagrams were also used by the great mathematician *Leonard Euler* and so these are also called Euler-Venn diagrams. We draw a rectangle or a circle, etc. to represent a set and the elements are then shown inside this rectangle (or circle) by dots as :



For example, in the following figure, the element a *belongs to* the set A while b *does not belong* to A.



1-4. Equal Sets

Consider the two sets

$$A = \{ 2, 3, 5, 7, 9 \}$$

and $$B = \{ 5, 7, 3, 2, 9 \}.$$

We observe that the elements in A are the same as the elements in B. But the elements of A are listed in an order different from the order of the elements in B. Thus A and B are only different names for the same set of elements as the order of the elements in a set is immaterial We say that such sets A and B are *equal* (or indentical), and write

$$A=B.$$

Two sets A and B are said to be identical (or equal) if each element of A is in B and each element of B is in A.

EXERCISE SET 1.2

Oral

1. Read the following :
   - (a) { Rama, Rahim, Richard, Ray }.
   - (b) $A = \{ 5, 15, 25, \ldots \}$.
   - (c) $B = \{ 100, 101, \ldots, 999 \}$.
   - (d) $a \in V$.
   - (e) $10 \in A$.
   - (f) $P = Q$.
   - (g) $R \neq T$.

2. State whether the following *statements* are true :
   - (i) 50 ∈ { Numbers divisible by 10 }.
   - (ii) Ganges ∈ { mountains in India }.
   - (iii) 7 ∈ { numbers greater than 6 }.
   - (iv) Mysore ∈ { States in India }.

3. Select a well-defined description for each set in the following :
   - (a) T = { April, June, September, November },
     - (i) a set of four months,
     - (ii) a set of happy months,
     - (iii) a set of months having only 30 days.
   - (b) $L = \{ s, e, t, l \}$;
     - (i) a set of four letters,
     - (ii) a set of three consonants and one vowel,
     - (iii) a set of letters in the word '*settle.*'

4. Which of the following sets are equal to one another ?

   $F = \{ a, g, k, q \}$; $G = \{ k, g, a, b \}$;

   $H = \{ k, q, g, a \}$; $L = \{ a, g, k, q, w \}$.

5. Is the set of odd numbers an infinite set ?

6. State whether the following sets are finite or infinite :

A = { Houses in Jaipur },
B = { All even numbers }.

Written

7. Express the following using symbols :
(i) The element t belongs to the set H.
(ii) The set consisting of Seeta, Susan, Nalini, Neela.
(iii) The set of the rivers of India.
(iv) Bombay does not belong to the set of capitals of the countries.
(v) The set M is the same as the set N.
(vi) A is the set consisting of 0, 2, 4, 6, and so on.
(vii) A set consisting of 0, 2, 4, 6, and so on to 100.

8. Write the following sets in Braces Form :
(i) the set of counting numbers less than 8,
(ii) the set of counting numbers between 6 and 10.

9. Write the following sets in the set-builder form :

B={ 3, 6, 9, 12, . . . }
C={ 0, 4, 8, 12, 16, . . . }.

10. A is the set of numbers ending in 5 and B the set of numbers ending in 0. Insert the symbol $\in$ or $\notin$ suitably :

(a) 25 ☐ A (b) 0 ☐ B (c) 30 ☐ B
(d) 50 ☐ A (e) 105 ☐ B (f) 500 ☐ A.

11. Given $A = \{a, b, c\}$, $B = \{e, f, g, h\}$ and $C = \{a, e, i, o, u\}$; insert the symbols $\in$ or $\notin$ suitably :

(i) a ☐ A (ii) a ☐ B (iii) a ☐ C
(iv) b ☐ A (v) b ☐ B (vi) b ☐ C
(vii) f ☐ A (viii) f ☐ B (ix) f ☐ C

12. Consider the sets A, B and C given in Exercise 11. Specify the element or elements having the following characteristics ·

(i) which belong to A and C ;

(ii) which belong to B and C ;

(iii) which belong to A, B and C.

1-5 Set and its Subsets

Consider the set S of boys listed below with marks obtained by them in a mathematics contest :

Name	*Marks*
Arvind	60
Yogesh	45
Raghu	52
Ronald	48
Jyoti	70
Kamalesh	59

Here S = {Arvind, Yogesh, Raghu, Ronald, Jyoti, Kamalesh}.

If D represents the set of boys, who have scored above 55 marks in the contest, who are the members of the set D ?

The members of the set D are Arvind, Jyoti and Kamalesh, and therefore

D = { Arvind, Jyoti, Kamalesh }.

If A represents the set of boys scoring above 50 marks in the contest, who are the members of the set A ?

The members of the set A are Arvind, Raghu, Jyoti and Kamalesh, and therefore

A = { Arvind, Raghu, Jyoti, Kamalesh }.

In the above examples, every element of D is an element of S, similarly every member of the set A is a member of the set S. In such cases, D and A are called *subsets* or parts of S.

A set P is a subset of Q, if and only if every element of P is an element of Q.

'A is a subset of S' is written symbolically as

$$A \subset S$$

It is also read as *'A is included in S'*, or *'A is contained in S'*. We can also express it as

$$S \supset A$$

Which is read as *'S is a superest of A'*, *'S contains A'* or *'S includes A'*

If C represents the set of boys, who have scored above 40 marks in the contest considered above, then

C={ Arvind, Yogesh, Raghu, Ronald, Jyoti, Kamalesh }.

Again, every member of C is also a member of S. Not only that, C and S have the same members, so C is a subset of S and C=S. Thus we see that S is a subset of S. In fact

Every set is a subset of itself.

When a subset of a set is not equal to the set, the subset is called a *Proper subset.*

What does $A \not\subset B$ mean ?

It means *'A is not a subset of B'* or *'A is not included in B'*.

Now consider the set of boys in your class who are 220 cm tall. Is there any boy of that height in your class ? There is no boy of that height. That means there are no members in the set;

the set of such boys is then said to be empty. When a set has no element in it, we say that the set is

empty, void or null.

The *void set* or *null set* is denoted by

$$\varnothing \text{ or } \{\ \}.$$

A set which has atleast one element is called a *non-empty* or *non-void set.*

Here are a few more examples of void sets.

(i) The set of women Presidents of India before 1970.

(ii) The set of numbers divisible by 7 and lying between 22 and 27

Now consider the set

$$S = \{a, b, d, e, l\}$$

and the sets

$$A = \{a, b, l\},$$

$$B = \{a, d, m, n\},$$

$$C = \{p\}.$$

Is $A \subset S$?

Every element of A is an element of S, so

$$A \subset S.$$

Is $B \subset S$?

There are some elements of B which do not belong to S.

For example,

$$m \in B \text{ but } m \notin S$$

$$n \in B \text{ but } n \notin S$$

so

$$B \not\subset S.$$

Is $C \subset S$?

There is an element p of C which does not belong to S. Since, $p \in C$ but $p \notin S$, so $C \not\subset S$.

A is not a subset of B, if A contains atleast one element which is not in B.

The void set is taken as a subset of every set.

Example 1 : The subsets that can be had from $\{1\}$ are

$$\varnothing \text{ and } \{1\}.$$

Example 2 : The subsets that can be had from $\{p, r, x\}$ are

$$\varnothing, \{p\}, \{r\}, \{x\}, \{p, r\}, \{p, x\}, \{r, x\}, \text{ and } \{p, r, x\}.$$

Note.—A set which contains all subsets under discussion is often called a **universal set.**

EXERCISE SET 1-3

Oral

1. Read the following :

(i) $p \in M$ (ii) $P \subset M$ (iii) $a \in \{a, b, c\}$

(iv) $\{a, b\} \subset \{a, b, c\}$ (v) $L \supset T$ (vi) $l \notin T$

(vii) $T \neq G$ (viii) $G = \varnothing$ (ix) $L = \{\ \}$

2. State whether the following statements are true :

(i) $\{a, b, c\} \subset \{a, b, c, d\}$

(ii) $\{a, b, c\} \subset \{a, b, d, e\}$

(iii) $\{a, b, c\} \subset \{a, b, c\}$

(iv) A set and its subset cannot be equal.

3. (i) Which set is the subset of all sets ?

(ii) Are the two sets $\{\ \}$ and $\{0\}$ the same ?

Written

4. Express the following using symbols :

   (i) A is a subset of M.

   (ii) The set of teachers is not included in the set of pupils in a school.

   (iii) The set consisting of a, e is a subset of the set of vowels.

5. List all the elements in the following sets :

   (i) $\{2, 4, 6, \ldots, 22, 24\}$.
   (ii) $\{4, 7, 10, 13, \ldots, 31\}$.
   (iii) $\{1, 3, 9, 27, \ldots, 729\}$.
   (iv) $\{1, 9, 25, \ldots, 169\}$.

6. Insert $\subset$ or $\not\subset$ suitably so that the resulting statements are true :

   (i) $\{p\} \square \{m, n, p\}$
   (ii) $\{p, q\} \square \{q, r, s\}$
   (iii) $\{p, q, r\} \square \{s, r, q, p\}$
   (iv) $\{p, q, s\} \square \{p, q, r, t\}$
   (v) $\{p, q, r, t\} \square \{t, q, p, r\}$
   (vi) $\{t, q, p, r\} \square \{p, q, r, t\}$.

7. Some of the following are pairs of equal sets and some are not Insert $=$ or $\neq$ suitably :

   (i) $\left\{\begin{matrix}\text{Months which do not have}\\ \text{either 30 days or 31 days}\end{matrix}\right\} \square \left\{\begin{matrix}\text{Months which begin}\\ \text{with the letter F}\end{matrix}\right\}$.

   (ii) $\left\{\text{Numbers which end in 2}\right\} \square \left\{\begin{matrix}\text{Numbers which are}\\ \text{divisible by 2}\end{matrix}\right\}$.

   (iii) $\left\{\text{Numbers which end in 0}\right\} \square \left\{\begin{matrix}\text{Numbers which are}\\ \text{divisible by 10}\end{matrix}\right\}$.

8. U represents the set of five students, whose names, ages and heights are given below :

Name	*Age*	*Height*
John	12 years	125 cm
Ahmed	11 years	123 cm
Mohan	12 years 2 months	120 cm
Tara	10 years	117 cm
Nargis	10 years	114 cm

Form the following subsets :

(i) L, the set of students whose ages are less than 11 years.

(ii) M, the set of students whose heights are between 115 cm and 124 cm.

(iii) G, the set of students whose ages are less than 11 years and whose heights are between 115 cm and 124 cm.

(iv) H, the set of students whose ages are 11 years and heights 125 cm

9. (i) Which set has only one subset ?

(ii) What members will be common to a set and its subset ?

10. (i) Form all the subsets that can be had from the set

$$\{ a, b \}.$$

(ii) Form all the subsets that can be had from the set

$$\{ e, f, g, h \}.$$

1-6 Intersection

You have seen, by now, that two sets may have common elements.

Take the two sets :

$$P = \{4, 8, 10, 12\},$$

$$Q = \{2, 4, 6, 8, 14, 20\}.$$

Are there some elements common to the sets P and Q ?

Yes, 4 and 8 are the elements common to P and Q.

A new set of elements $\{4, 8\}$ is formed by taking together the elements common to P and Q and we call it *'intersection'* of P and Q, which is written symbolically as

$$P \cap Q$$

Therefore, $P \cap Q = \{4, 8\}$.

$P \cap Q$ is read as *'P intersection Q'* or sometimes as *'P Cap Q'*.

To represent intersection of two sets in Venn diagrams, we take a set

$U = \{a, b, c, d\}$ and the following subsets of it :

$A = \{a\}$

$B = \{d\}$

$C = \{b\}$

$D = \{c\}$

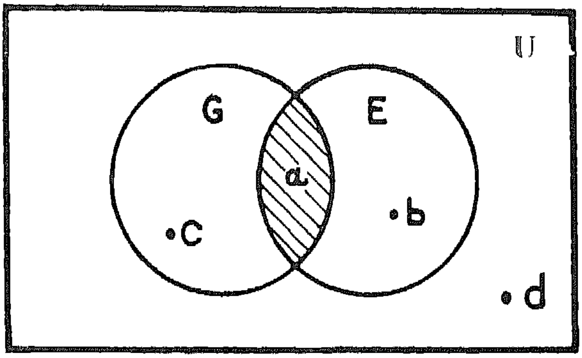
$E = \{a, b\}$

$F = \{b, c\}$

$G = \{c, a\}$

$H = \{a, b, c\}$.

The intersection of E and G is represented by the elements or element contained in the portion common to the two circles as shaded in the figure given below :

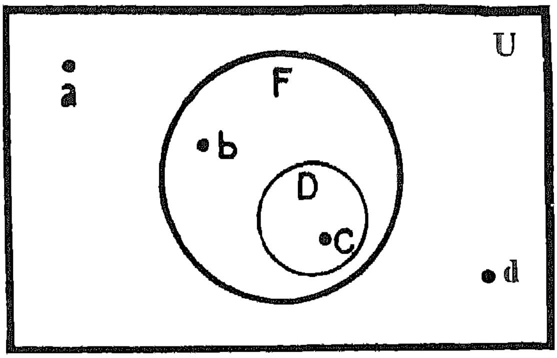


Thus $E \cap G = \{a, b\} \cap \{c, a\} = \{a\}$.

What is $F \cap D$?

$$F \cap D = \{b, c\} \cap \{c\} = \{c\}.$$

Also D is a subset of F. This is shown in the Venn diagram as

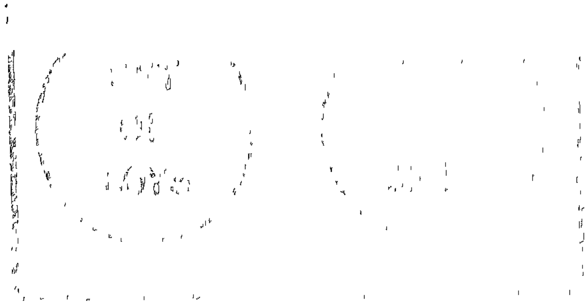


What will be the intersection of B and F ?

You see that there is no common element between B and F; therefore, the intersection of B and F is the empty set.

So $B \cap F = \varnothing$.

Sets which have no common members or elements are called *disjoint sets.* The set of boys and the set of girls in a class are disjoint and are represented by the non-intersecting circles as :



1-7 Union

Consider again the sets P and Q :

$$P = \{ 4, 8, 10, 12 \}$$

$$Q = \{ 2, 4, 6, 8, 14, 20 \}.$$

Let us now form the set whose members belong either to P or to Q or to both.

It is the set

$$\{ 2, 4, 6, 8, 10, 12, 14, 20 \}$$

We call it the *union* of P and Q, which is written symbolically as :

$$P \cup Q.$$

Therefore,

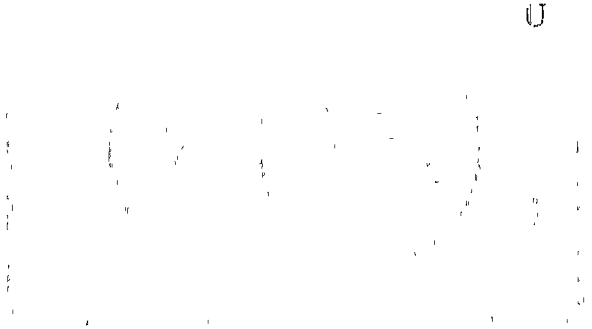
$$P \cup Q = \{ 2, 4, 6, 8, 10, 12, 14, 20 \}.$$

'P ∪ Q' is read as *'P union Q'* or sometimes as 'P cup Q.' Thus 'P union Q' is the set whose members belong either to P or to Q or to both.

Also considering the sets U, A, B, C, D, E, F, G and H (Given in Art. 1-6) we have

$$E \cup G = \{ a, b \} \cup \{ c, a \} = \{ a, b, c \}.$$

In Venn diagrams this is shown by the shaded portion as given below .



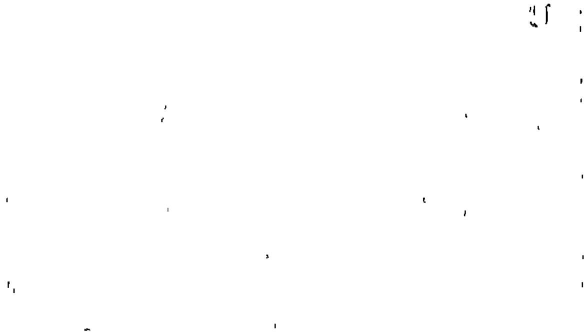
Again since

$$F = \{ b, c \},$$

$$C = \{ b \},$$

$$\therefore \; F \cup C = \{ b, c \}.$$

This is shown by the shaded portion as given below :



EXERCISE SET 1-4

Oral

1. Read the following :

   (i) $M \cap N$ (ii) $H \cup G$.

2. Fill up the blanks suitably with symbols for *intersection* or *union* :

   (i) $\{a\}=\{a, b, e\} \square \{m, n, a\}$.
   (ii) $\{a, b, c\}=\{a, b\} \square \{b, c\}$.
   (iii) $\{a, b\}=\{a\} \square \{a, b\}$.
   (iv) $\{a, b\}=\{a, b\} \square \{a, b, c\}$.

3. Give the intersection of the following pairs of sets in the braces form :

   (i) $\{2, 5, 9\}$, $\{2, 5, 7\}$,
   (ii) $\{5, 7, 8, 11\}$, $\{7, 4, 8, 11, 5\}$,
   (iii) $\{1, 2\}$, $\{6, 12\}$,
   (iv) $\{a, b, c\}$, $\{b, c, f\}$.

4. Give the union of the following pairs of sets in the braces form :

   (i) $\{4, 6, 8\}$, $\{2\}$.
   (ii) $\{4, 6, 8\}$, $\{\ \}$.
   (iii) $\{4, 6, 8, 13\}$, $\{4, 6, 13\}$.
   (iv) $\{p, q, r\}$, $\{r, q, p\}$.

Written

5. Express the following using symbols :

   (i) D intersection G.

   (ii) The union set of E and F.

6. Form new sets by forming the *union* of the following sets :

   (i) $\{3, 5, 7\}$, $\{2, 4, 5\}$. (ii) $\{1, 2, 3\}$, $\{2, 3, 4\}$.
   (iii) $\{a, b, c\}$, $\{b, c, a\}$. (iv) $\{a, b, c\}$, $\{a, b\}$.
   (v) { Ram, Mohan }, { Ram, Sita, Nutan }
   (vi) $\{5\}$, $\{\ \}$.

7 Form new sets by finding the *intersection* of the following :

   (i) $\{1, 2, 3\}$, $\{3, 2\}$ (ii) $\{a, b, c\}$, $\{c, a\}$.
   (iii) $\{1, 2, 3\}$, $\{4, 5\}$ (iv) $\{a, b, c\}$, $\{a\}$.
   (v) $\{4, 7\}$, $\{\ \}$ (vi) $\{1, 0\}$, $\{10\}$.
   (vii) { Ram Sita, Mahesh }, { Ram, Mohan, Mahesh}.

8. Find the *intersection* of the following :

   (i) $\{l, t, u, v\}$ and $\{u, p, q, t, s, l\}$.
   (ii) $\{7, 9, 11\}$ and $\{9, 7\}$.

9. Find the *union* of the following :

   (i) $\{5, 7, 4, 3\}$ and $\{1, 0, 6\}$.
   (ii) $\{a, b, c, d\}$ and $\{b, e, d\}$.

10. Consider the four sets A, B, C and D specified below :

$$A = \{a, b, c\},$$
$$B = \{b, c, d\},$$
$$C = \{c, d\},$$
$$D = \{a\}.$$

Find : (i) $A \cap B$ (ii) $B \cap C$ (iii) $C \cap D$ (iv) $A \cup B$
(v) $B \cup C$ (vi) $A \cup D$ (vii) $A \cup A$ (viii) $A \cap A$.

11. Given
$$\begin{aligned} U &= \{1, 2, 3, 4, \ldots, 12\}, \\ A &= \{2, 4, 6, 8, 10, 12\}, \\ B &= \{1, 3, 5, 7, 9, 11\}, \\ C &= \{3, 6, 9, 12\}, \\ D &= \{2, 3, 4, 5, 6, 7\}; \end{aligned}$$

write the following sets in the braces form :

(i) $A \cup U$	(ii) $B \cap U$	(iii) $C \cup U$
(iv) $D \cap U$	(v) $U \cup B$	(vi) $U \cap C$

12. Insert the symbol $\cap$ or $\cup$ suitably in the following .

(i) $\{a, k, d, e\} \;\square\; \{g, d, k\} = \{d, k\}$.

(ii) $\{5, 11, 6, 4, 3\} \;\square\; \{11, 6, 4, 3, 2\} = \{11, 6, 4, 3, 2, 5\}$.

(iii) {Ram, Ravi, Raghu} □ {Raghu, Ravi}
= {Ram, Ravi, Raghu}.

(iv) {Ram, Ravi, Raju} □ {Raju, Ram} = {Ram, Raju}.

13. Insert appropriate symbols, so as to make the following true :

(i) $\{8, 9\} \;\square\; \{5, 6, 9, 3, 8\}$.

(ii) $\{7\} \;\square\; \{8, 9\}$.

(iii) $\{1, 2, 4\} \;\square\; \{3, 5\} = \{\;\;\}$.

14. Draw Venn diagrams for the following :

(i) $\{8, 5, 3, 2\} \cap \{5, 3, 1, 4\}$.

(ii) $\{a, b\} \cup \{c, d, e\}$.

15. Use Venn diagrams to answer the following :

(i) If $A \subset B$, what is $A \cap B$? What is $A \cup B$?

(ii) If $A \cap B = A$, which of the following statements is true :

$B \subset A$ or $A \subset B$?

(iii) If $A \cup B = A$, which of the following statements is true ·

$B \subset A$, $A \subset B$?

(iv) If $A \cap B = C$, which of the following statements is always true :

$$A \subset C \quad , \quad C \subset A ?$$

16. (i) If $a \in A \cap B$, does $a \in A$ and $a \in B$?
(ii) If $a \in A \cup B$, does $a \in A$ and $a \in B$?
(iii) If $A \subset B$ and $B \subset A$, what can you say about A and B ?

17. If $A = \{1, 2\}$, $B = \{2, 3, 4\}$, $C = \{2, 3, 4, 5, 6\}$, form the following new sets :
(i) $(A \cap B) \cap C$. (ii) $A \cup (B \cup C)$.
(iii) $A \cap (B \cap C)$ (iv) $(A \cup B) \cup C$

18. If $A \subset B$ and $B \subset A$, find
(i) $A \cap B$ (ii) $A \cup B$.

*19. List the set of boys in your class whose names begin with G and the set of girls whose names begin with G. Can you now, making use of these two sets, list the set of pupils in your class whose names begin with G ? What operation have you made use of, union or intersection ?

*20. Give an example for two non-empty disjoint sets ?
What is their intersection ?

*21. Take any two sets A and B, find
(i) if $A \cup B$ is different from $B \cup A$?
(ii) if $A \cap B$ is different from $B \cap A$?

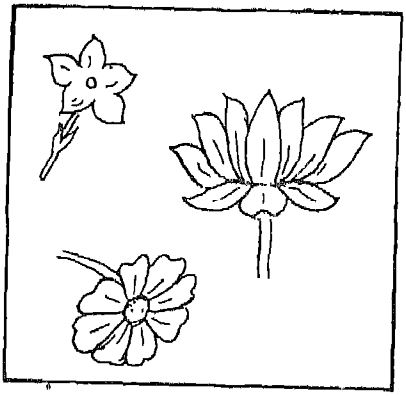
*22. Take any three sets P, Q, and R. Form the following new sets :
(i) $(P \cap Q) \cap R$. (ii) $P \cap (Q \cap R)$
(iii) $(P \cup Q) \cup R$. (iv) $P \cup (Q \cup R)$.
(v) $(P \cap Q) \cup R$. (vi) $(P \cup Q) \cap R$.

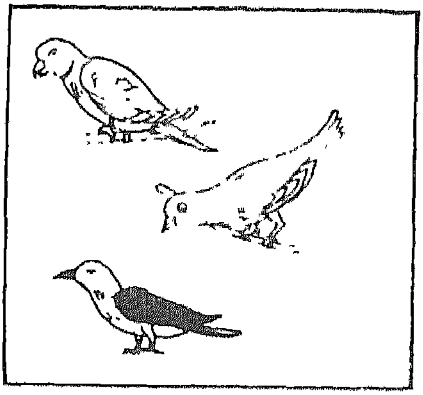
2

THE SET OF WHOLE NUMBERS : W

2-1 Comparison of Sets

Consider the two sets pictured below :



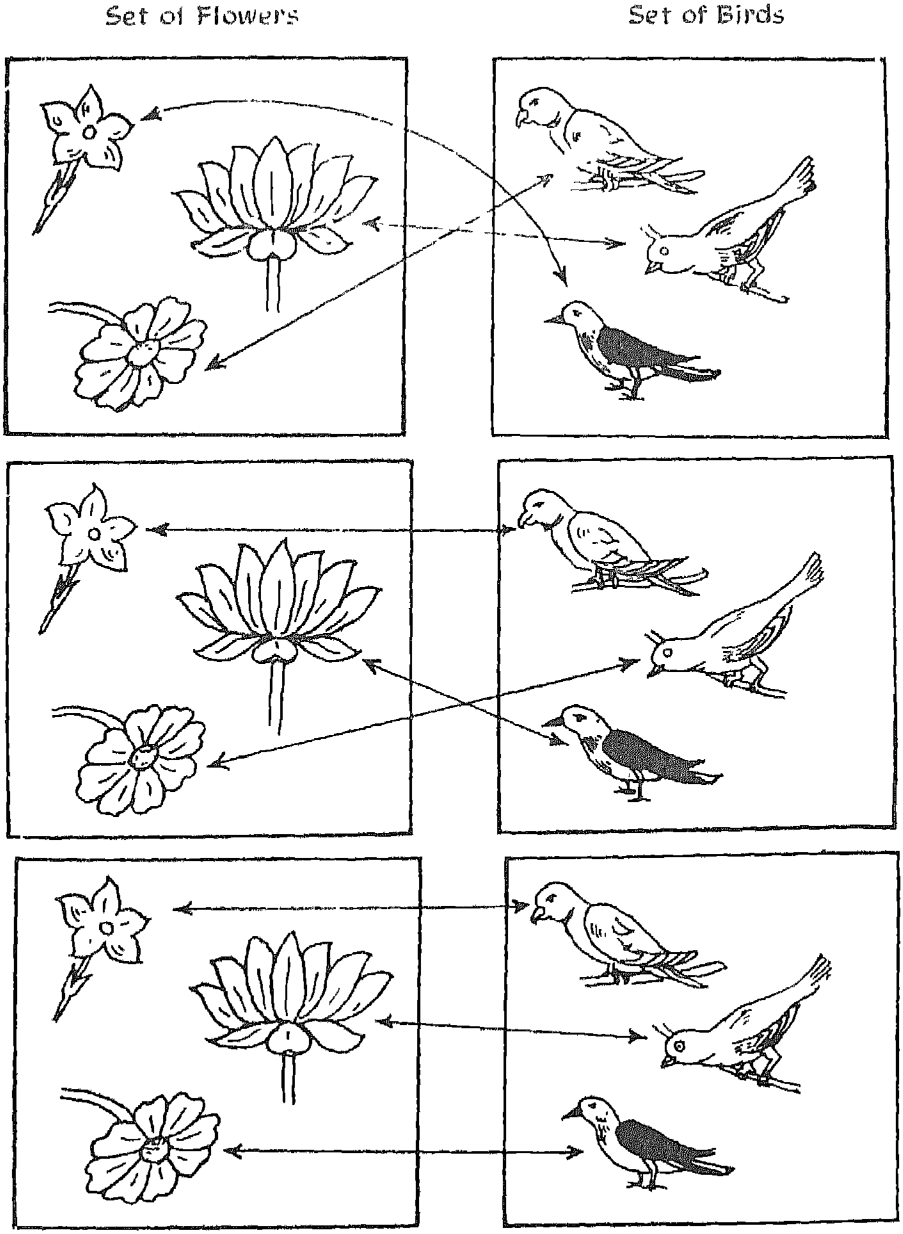


One is a set of flowers and the other is a set of birds. Can they be compared ? There seems to be nothing in common between them; but if you examine them carefully, you find that one set has *as many* elements *as* the other set. How have you found it ? By counting, which you learnt in earlier classes ?

Now suppose that you are not to count. How will you then show that one set has as many elements as the other ? For this, we consider an example which may be very familiar to you. A milkman in a village, gives some measures of milk everyday to a household and gets payment for the amount of milk given, at the end of every month. He marks as many lines (or thick dots) as the measures of milk he gives everyday, on a wall or at some other place. The set of lines (or dots) in any month can be made to correspond in a *one-to-one* way to the measures of milk given by him during that month. In other words we say that there are as many elements in the set of lines (or dots) as there are in the set of measures of milk given by him.

This method adopted by the milkman is called comparison through *one-to-one correspondence.* You will see, as you learn more and more of mathematics, that it is found very useful by mathematicians.

Now can you use this technique of one-to-one correspondence in comparing the set of flowers and the set of birds ? The comparison can be done in a number of ways. For example :



Let us name the set of flowers, F and the set of birds, B. Take any matching diagram. Now, to how many members of B is every member of F associated ? Is any member of B associated with more than one member of F ?

To the first question your answer is *one and only one* and to the second question, your answer is *no*. Thus the two sets are said to be in *one-to-one correspondence*, if every member of either set can be associated with one and only one member of the other set.

When any two sets have their elements in one-to-one correspondence, one set has as many elements as the other

If there are two sets $\{a, b\}$ and $\{p, q\}$, they can be put into one-to-one correspondence as

$$\begin{array}{c} \{\, a, \; b \,\} \\ \updownarrow \;\; \updownarrow \\ \{\, p, \; q \,\} \end{array}$$

and are said to *match* each other. Further since these sets have their elements in one-to-one correspondence, they have the same number of elements.

What happens, if two sets do not match as shown below ?

$$\begin{array}{lllllllllllll} A = \{ & a, & e, & i, & o, & u \} & & & & & & \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & & & & & \\ B = \{ & b, & c, & d, & f, & g, & h, & j, & k, & l, & m, & n \}. \end{array}$$

We find that the set B has more elements than the set A.

The set of your ears can be matched with the set of your eyes, but neither set can be matched with the set of your fingers on the right (or the left) hand.

Sets which can be matched one-to-one are called *equivalent* or *equipollent sets*.

If A and B are equivalent sets, it is denoted symbolically as

$$A \longleftrightarrow B$$

Accordingly

$$\{a\},\quad \{c\},\quad \{\square\},\quad \{\text{Ram}\},\quad \{+\},\quad \{\Delta\}$$

are all equivalent sets, but

$$\{a, b\},\quad \{a, b, c\},\quad \{\square, \Delta, \text{O}\},\quad \{\text{Ram}\}$$

are not equivalent sets.

EXERCISE SET 2-1

Oral

1. Give three examples of equivalent sets, taking the parts of your body.

2. Give two examples of non-equivalent sets, taking again the parts of your body.

3. Say if the following are equivalent sets or not :
   - (i) the set of classes and the set of class-teachers in your school.
   - (ii) the set of brothers and the set of sisters in your home.
   - (iii) the set of countries and the set of their capitals.
   - (iv) the set of schools and the set of their headmasters.

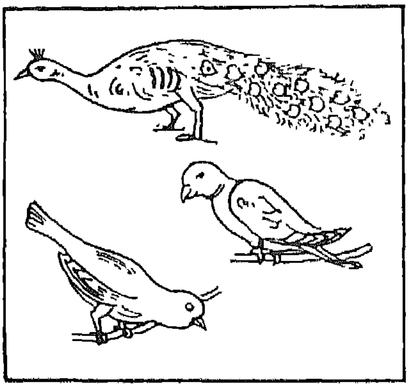
Written

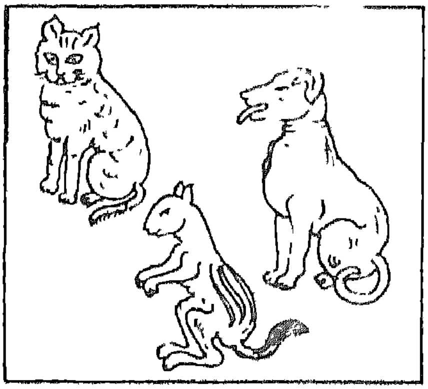
4. In an elementary school, boys and girls were asked to stand alternately in a row in an entertainment programme. The row started with a boy. If the set of boys and the set of girls were equivalent, who will be the last person in the row—a boy or a girl ?

5. Pick out the equivalent sets from the following ?

$$A = \{5, 6, 11, 14\}$$
$$B = \{a, b, c, d, e\}$$
$$C = \{a, b, c, d, r, t\}$$
$$D = \{4, 6, 11, 13,\}$$

6. Study the diagrams given below and say which of them can be displayed by one-to-one correspondence :





Delhi

Kanpur

Jaipur

Madras

Himalayas

Vindhyas

Satpura

*7. Draw all the possible matching diagrams for the following pairs of sets :

(i) $\{m, c\}$ and $\{\square, \triangle\}$

(ii) $\{a, b, c\}$ and $\{\square, \triangle, \bigcirc\}$

(iii) $\{a, b, c, d\}$ and $\{\square, \triangle, \bigcirc, \times\}$

*8. If a set A is equivalent to a set B and the set B is equivalent to a set C, are the sets A and C equivalent ? Give examples to justify your answer.

2·2 The Whole Numbers

In the last section, you have learnt that if the elements of one set can be put in one-to-one correspondence with the elements of another set, the two sets are called equivalent.

Consider the sets

$$A = \{ \triangle, \square, \diamond, \circ, \times \}$$

and $$B = \{ p, q, l, t, v \}$$

Or

$$C = \{ \infty, \emptyset, \smile, \times \}$$

and $$D = \{ >, <, \vee, \wedge \}.$$

We observe that $A \leftrightarrow B$ and $C \leftrightarrow D$.

If two sets are equivalent, we shall say that they have the same *cardinal number*. If we write n (A) for cardinal number of set A, then

$$n(A) = n(B) \text{ iff. } \quad A \leftrightarrow B.$$

Note :—Iff. is read as 'if and only if'

This fact that $A \leftrightarrow B$, when A and B are finite is also mentioned by saying that A and B have the *same number of elements* [Remember not same elements]. But 'How many' ? To answer this question, we associate some symbol with each different cardinal number and also give it some name. This name represents also the number of elements in a set.

Naturally to assign these names and symbols we consider the familiar set of symbols

$\{ 1, 2, 3, \ldots \}$ called one, two, three, etc.

We associate the symbol 'p' to the cardinal number of all sets equivalent to $\{ 1, 2, 3, \ldots, p \}$; thus

$n\{1\} = 1$, named one,
$n\{1, 2\} = 2$, named two.
$n\{1, 2, 3\} = 3$, named three, etc.

The following table gives the cardinal number name of some sets and also symbols (numerals) representing that cardinal number name.

Equivalent sets	Cardinal number name	Symbol
$\{a\} \leftrightarrow \{\triangle\} \leftrightarrow \{\text{Ram}\} \leftrightarrow \{1\}$	one	1
$\{a, b\} \leftrightarrow \{\triangle, \diamond\} \leftrightarrow \{\text{Ram, John}\} \leftrightarrow \{1, 2\}$	two	2
$\{a, c, d\} \leftrightarrow \{\triangle, \diamond, \text{O}\} \leftrightarrow \{\text{Ram, John, Lala}\} \leftrightarrow \{1, 2, 3\}$	three	3
$\{a, c, d, e\} \leftrightarrow \{\triangle, \square, \diamond, \text{O}\} \leftrightarrow \{\text{Ram, John, Lala, Mali}\} \leftrightarrow \{1, 2, 3, 4\}$	four	4
$\{a, c, d, e, g\} \leftrightarrow \{\triangle, \square, \diamond, \text{O}, \neq\} \leftrightarrow \{\text{Ram, John, Lala, Mali, Karim}\} \leftrightarrow \{1, 2, 3, 4, 5\}$	five	5

and so on.

So the numbers one, two, three, etc, which you learnt in earlier classes are the *common properties* of equivalent sets. One, two, three, etc. is one way of naming the cardinal numbers. There are other ways of naming them; they vary from language to language; in Hindi, the names run as follows :

ek, *do*, *teen*, and so on.

In Tamil, they are

onru, *erandu*, *moonru*, etc.

In Sanskrit they are *aekam*, *dwee*, *threeni*, etc. In Japanese, they are ichi, ni, san, etc. But they represent the same sequence of cardinal numbers.

Order Relation

By comparing sets, we can find the order relation between any two cardinal numbers.

Since

$$\{a\} \subset \{a, b\},$$

the cardinal number of $\{a\}$ is less than the cardinal number of $\{a, b\}$. That is, 1 is less than 2. In symbols, we can write

$$1 < 2,$$

where $<$ is read as '*is less than*'.

Again

$$\{a, b\} \subset \{a, b, c\}.$$

So the cardinal number of $\{a, b\}$ is less than the cardinal number of $\{a, b, c\}$, that is

$$2 < 3.$$

Similarly we can show that

$$3 < 4$$
$$4 < 5$$

and so on.

So there is an order in the sequence of the cardinal numbers. This is the basis of counting. Now we are in a position to understand and explain the process of counting with the help of which we answer the question 'How many elements are there in a set'.

Suppose we want to find out how many elements are there in the set

$$P = \{k, l, g, t\}.$$

One way is to match it with each sample set associated with a cardinal number and once the sample set which matches one-to-one with P is found, the cardinal number of P is the same as the cardinal number associated with the sample set.

The set P matches one-to-one with the sample set $\{a, c, d, e\}$ of table (page 32) whose cardinal number is 4. So the cardinal number of P is 4.

To fix up the sample set, we make use of ordered sequence $\{1, 2, 3, 4, \ldots\}$.

Take the sample set $\{1, 2, 3, 4\}$. Match the set P with it. We find they are matched one-to-one as seen below :

$$\begin{array}{cccccccccc}
\{ & k, & l, & g, & t & \} \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \\
\{ & 1, & 2, & 3, & 4 & \}.
\end{array}$$

The number of elements in either set is 4. You see also that 4 is the last element in the set of numbers. This is how man started counting.

To count the elements in a given finite set means to match the elements of the set with the elements of the ordered set of counting numbers and the last matched element in the ordered set gives the number of elements of the given set.

The counting numbers are also called natural numbers.

Earlier in chapter I, you have learnt about the empty set. Now what is its cardinal number ? The cardinal number of the empty set is named *zero* and written *0*.

The set of numbers starting with 0 as $\{0, 1, 2, 3, \ldots\}$ is called the set of *whole numbers* and denoted as W.

The set of natural numbers has another important use. When you are told that there is a magic square in this book, what question do you ask ? You would like to know *on what* page it is. This is different from knowing the number of pages in the book. If you are told that the magic square is on the fortieth page, the number

40 on that page shows you the location of that page relative to other pages. Natural numbers when used to specify the elements of a set in a particular order are called *ordinal numbers.*

It is easy to make out from the context if a number is in the cardinal or ordinal sense.

EXERCISE SET 2-1
2-2

Oral

1. Read the following :

(i) $M \longleftrightarrow T$. (ii) $P \longleftrightarrow Q \longleftrightarrow S$.

(iii) $4 < 8$. (iv) $M \subset N \subset L$.

(v) $1 < 5 < 7$.

2 Find the cardinal number of each of the following sets :

(i) $\{ h, t, v, m, n, u \}$.

(ii) $\{ g, p \}$.

3. The order of elements in a set M is as shown below :

$$M = \{ g, p, r, a, k, t, u \}.$$

(i) What is the third element from the right ?

(ii) What is the third element from the left ?

(iii) Which element will have the same ordinal number, whether counted from the right or from the left ?

Written

4. Rewrite the following using symbols :

(i) the two sets T and H are equivalent;

(ii) the three sets D, E and F are equivalent;

(iii) Nine is greater than four;

(iv) A is a subset of B and B is a subset of C.

5. State whether the following statements are true :

   (i) Two sets having the same cardinal number are equivalent.

   (ii) Only equal sets have the same cardinal number.

   (iii) Sets which have the same cardinal number cannot be matched one-to-one.

   (iv) Every element of the set of natural numbers :

   $$\{1, 2, 3, \quad \}$$

   is an element of the set of whole numbers :

   $$\{0, 1, 2, 3, \ldots\}.$$

   (v) The set of whole numbers is a subset of the set of natural numbers.

   (vi) The set of natural numbers and the set of whole numbers are infinite sets.

6. Complete the following :

   (i) Zero is the cardinal number of.. ... . .. set.

   (ii) The cardinal number of a set K is greater than the cardinal number of a set L, if the set K has ......elements than the set L.

7. Find all the subsets of $\{a, b, c, d\}$ and classify them in equivalent sets.

2-3 Decimal Numerals

You have seen by including another element, yet another element and so on that we get a sequence of sets such that each is a subset of the next set, as shown below :

$$\{\ \} \subset \{a\} \subset \{a, b\} \subset \{a, b. c\} \subset \{a, b, c, d\}$$

and so on.

In turn the cardinal numbers associated with these sets form a sequence. Let us write the cardinal number associated with a set A briefly as n (A). If the set is given as { p, q }, then we write the cardinal number associated with it as n ({ p, q }). Now

$$n(\{\ \}) = 0$$
$$n(\{a\}) = 1$$
$$n(\{a, b\}) = 2$$
$$n(\{a, b, c\}) = 3$$

and so on.

Thus we have

$$n(\{\ \}) < n(\{a\}) < n(\{a, b\}) < n(\{a, b, c\})$$
$$< n(\{a, b, c, d\}) \text{ and so on,}$$

or $0 < 1 < 2 < 3 < 4$ and so on.

We reach a stage, when given a number we can always think of the next number, i.e., its *successor.*

1 is the successor of 0,

2 is the successor of 1,

3 is the successor of 2,

4 is the successor of 3,

and so on.

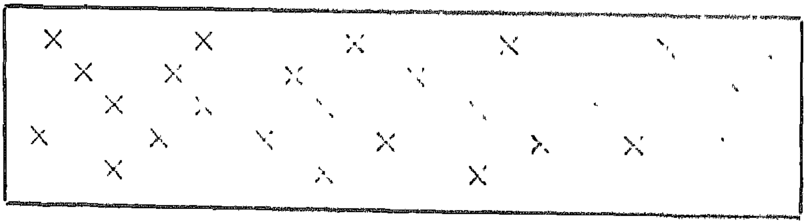
As civilisation progressed, man felt the need for bigger and bigger numbers and faced two problems : *How to name them* and *how to record them.* It took many centuries for man to solve these problems to his full satisfaction.

History records various attempts in different parts of the world to solve the problems. But they were all partial solutions. A complete solution to these problems emerged in India atleast two thousand years ago.

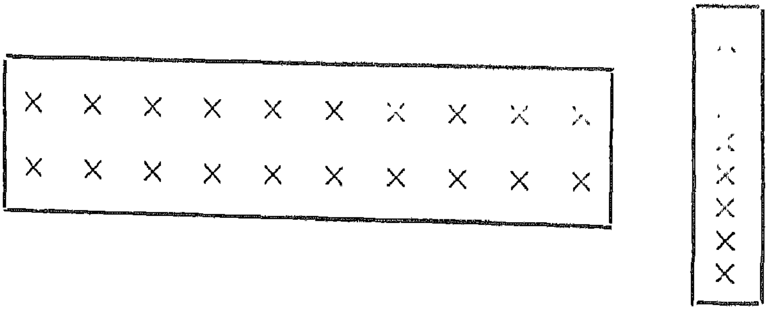
To give different names and different symbols, for the numbers as they come in succession will be tedious and unmanageable. So the idea of grouping dawned on man. Now all over the world the grouping is by tens and ten is called the base of this system. But grouping was tried differently in the past, before '*base ten*' system came to be accepted universally.

There was a time when the Chinese used 2 as a base. Some people living along the banks of Euphrates used 3 as base and some people on the Nile used 5 as base. The people on the Nile appear to have got the clue from five fingers on each hand; many people thought of fingers on both the hands and started using 10 as the base. By counting fingers and toes, 20 was also used as the base by the early inhabitants of America. The largest base used was 60 and that was by the Babylonians.

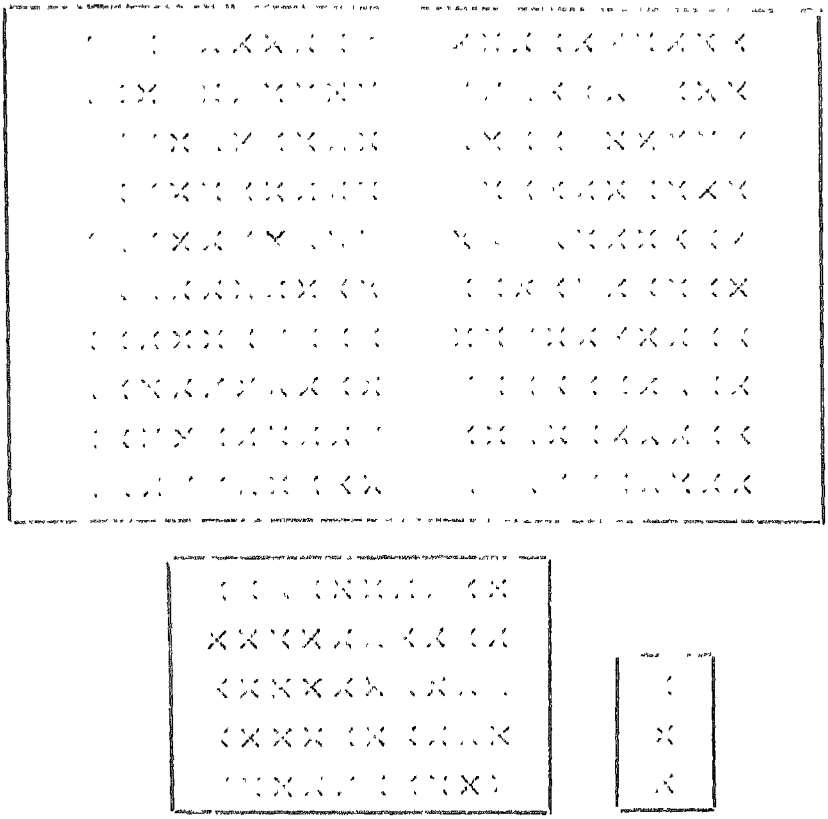
Suppose you are asked to find the number of crosses given below. What do you do ?



They are grouped into tens first; then the number of tens of them and the number of remaining ones are read. That gives the number of crosses as shown below :



There are two groups of ten crosses and eight single crosses or twenty eight crosses in all. If more tens are got such that they can be grouped to have tens of them, we form groups of *ten tens* or *hundreds*. See the illustration given below ·



The number of crosses consists of two hundreds, five tens and three ones, that is, two hundred fifty three.

If hundreds can be grouped further in tens, ten hundreds or thousands are got. This process of grouping can be taken to any stage. Thus, you see that at every stage in the process, grouping is done by tens. The system of numerals employed here to represent numbers is called the *decimal notation* as the base is ten. It is called the Hindu System as the Hindus were the first people to make use of it successfully.

Having given number-names, the next question is how to record them. In 'base ten' system, only ten basic symbols, viz. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are needed to write any number, e.g., a number which consists of 4 thousands, 7 tens and 8 units is represented by

4078.

The symbol 0, used here to represent absence of hundreds has been in use in our country from very early times. The original name in Sanskrit of the symbol is *Shunya* (empty), which assumed the present name *'Zero'* via the Arabs.

Zero along with its symbol which was used to serve as place holder was also taken as a number. It is the first number in the set of whole numbers, giving the cardinal number of the empty set.

The operations involving zero were defined in Hindu books of the first centuries of the Christian era.

In a numeral with more than one digit, every digit has a *place value* as well as the *face-value.* For example in 4078 the place value of 7 is 70, but its face-value is 7.

We recall here the place value units in current use in English as well as in Sanskrit and Hindi :

	English	Hindi	Sanskrit
1	one	aek	ekam
10	ten	das	dasam
100	hundred	sau	satam
1000	thousand	hazar	sahasram
10000	ten thousand	das hazar	ayutam
100000	hundred thousand	lakh	laksham
1000000	million	das lakh	niyutam
10000000	ten million	karod	koti
100000000	hundred million	das karod	prayutam

In order to facilitate the reading of numbers having several digits (in their decimal notation, more strictly) commas are sometimes employed grouping 2 or 3 consecutive digits, for example

21, 42, 78, 63, 481.

EXERCISE SET 2.3

Oral

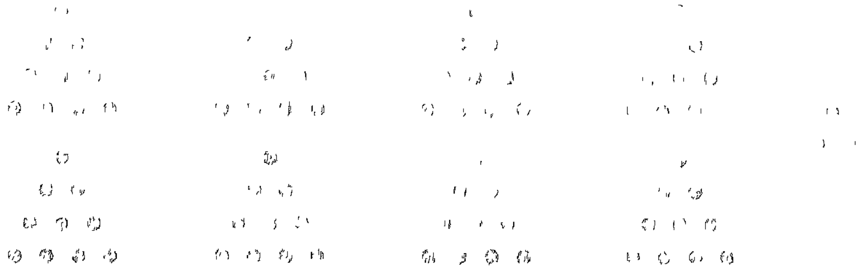
1. Read the following :

(i) n ({ *p*, *l*, *t*, *m*, *r*, *s* }).
(ii) n (A) < n (B).
(iii) 48, 21, 65, 719.

2. Give the place value of 3 in each of the following numbers :

(i) 73841. (ii) 3846851. (iii) 1000309.

3. Give the number represented by the following set of dots :



4. What is the number of which 1 is the successor in the sequence of whole numbers ?

Written

5. Rewrite the following in symbols :

(i) The cardinal number of the set consisting of *v*, *e*, *f*, *b*, and *c*

(ii) The cardinal number of the set P is less than the cardinal number of the set G.

(iii) Ninety crore, eight lakh, three thousand three.

6. Complete the following :

(i) 4814 consists of ▭ tens and ▭ ones.
(ii) 4814 consists of 48 ▭ and 14 ▭.

7. Insert >, < or = suitably in the following statements to make them true :

   (i) $n(\{l, k, v, t, u, r\})$ □ $n(\{a, b, c, d, l, k, t, v\})$.

   (ii) $n(\{p, q, r, s, h, l, m\})$ □ $n(\{7, 5, 3, 0, 1, 2\})$.

8. Complete the following :

   (i) The place value of 9 in 190723 is □.

   (ii) The face-value of 7 in 190723 is □.

   (iii) If two sets are equivalent, their cardinal numbers are □.

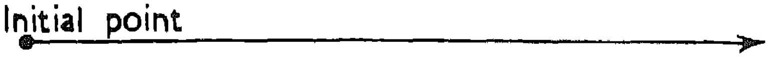
9. Answer the following, giving examples :

   (i) If two numbers are such that the first number is the successor of the second, which is the greater of the two numbers ?

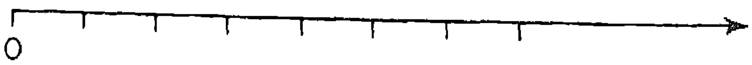
   (ii) If two numbers are such that the first number is greater than the second number, can you always say that one of them is the successor of the other ?

2-4 Number-Ray and Order Relation

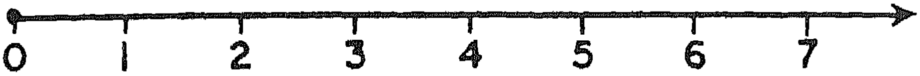
The set of whole numbers can be represented as a set of points on a ray. You know that a ray begins at a point called the initial point and extends endlessly in one direction (to the right in the figure) as shown below



The set of whole numbers begins with the number 0. Let us associate the initial point with 0. Choose any unit of length and mark off points on the ray starting from the initial point at



unit intervals. Now we get a set of equally-spaced points starting at the point marked 0 and extending endlessly to the right. Let these points to the right of 0 be associated with 1, 2, 3, 4, 5 and so on.



You find that the set of whole numbers is in *one-to-one* correspondence with this set of points on the ray. This ray is called the *number ray*. This number ray is also called the *graph* of the set of whole numbers. Each number is called the *coordinate* of the corresponding point.

The graph shows that the successor of a whole number is represented by the point immediately to the right of the point representing the whole number.

Take whole number, say 4. The numbers to its right are all greater than it. The numbers to its left are all less than it. This is true of any whole number except zero.

What is the number whose successor is zero ? There is no whole number whose successor is zero.

Think of any two whole numbers. If you think of the same two numbers, then they are equal. If you think of different numbers, one on the right is greater than the one on the left of it on the ray. Suppose the two numbers you have thought are 7, 7. Then $7 = 7$. If the two numbers you have thought are 8, 6. Then 8 is greater than 6, written symbolically as

$$8 > 6$$

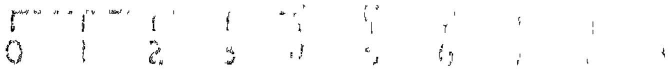
We can also say that 6 is less than 8 and write it symbolically as

$$6 < 8.$$

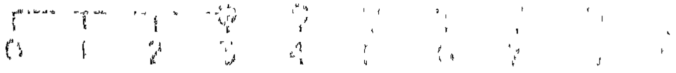
The symbol $>$ is read as '*is greater than*' and the symbol $<$ is read as '*is less than*'.

If two numbers have several digits, the number with more digits is greater than the number with less digits, e.g., 1043 > 998.

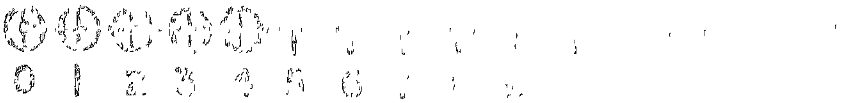
Between any two whole numbers, provided one is not the successor of the other, there is a *finite set* of whole numbers. For example, between 3 and 7, there are 4, 5, 6 forming a finite set The graph of the set of numbers between 3 and 7 can be shown as follows :



Hollow dots are placed against 4, 5 and 6. Again, the graph of numbers greater than 2 can be shown as follows :



Hollow dots are placed against 3, 4, 5 and so on. Similarly the graph of whole numbers less than 5 is as shown below :



Hollow dots are placed against 4, 3, 2, 1 and 0.

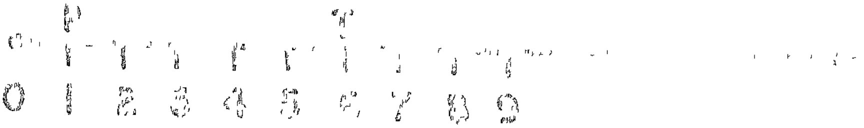
EXERCISE SET 2-4

Oral

1. Read the following :

(i) 9 > 2 (ii) 3 < 8 (iii) 1 > 0.

2. Give the coordinates of points named P and T on the number ray :



3. Which of the following statements are true :

(i) $7 > 9$ (ii) $8 > 0$ (iii) $1 > 1$ (iv) $2 < 10$.

4. Complete the following statements so that they are true :

(i) The set of whole numbers less than 5 consists of

______ ______. ______. ____________________.

(ii) Given any two whole numbers, the greater number is represented on the number-ray to the ________ of the smaller number.

(iii) The whole numbers greater than a given whole number form ___ _ ___ ______ set.

(iv) The whole numbers less than a given whole number form _ _ _ _set.

Written

5. Draw the graph of the set of whole number between 1 and 10.

6. Draw the graph of the set of whole numbers less than 6.

7. Draw the graph of the set of whole numbers greater than 8.

8. Compare the following pairs of numbers and state whether the first is greater or less than the second. Insert the suitable sign > or < :

First		Second
37	☐	32
45	☐	48
37	☐	29
41	☐	38

9. What whole numbers if put in the frames of the following will give true statements ? Put down your answer for each in the form of a set.

(i) $\square < 11$.

(ii) $8 < \square < 13$.

(iii) $\square > 6$.

(iv) $\square = 15$.

*10. Consider two whole numbers such that the first is greater than the second. Consider a third whole number such that the second is greater than the third. Compare the first and the third and state how they are related. Try a number of examples and give a general statement.

2-5 Addition in W

You have already been explained that each whole number can be associated with certain finite set as its cardinal number. Now we shall try to explain the operation of addition in W by giving its definition with the help of cardinal numbers of disjoint sets.

Let us recall how you learnt to add any two whole numbers, say 5 and 4 in your lower classes. You counted 5 finger-tips on your fingers or took a set of 5 balls. Then you counted another 4 finger-tips or took another set of 4 balls and counted the finger-tips or balls in the combined set. Naturally no finger-tips or balls was common to the first two sets. Thus we may define :

If there are two sets A and B and if

$$A \cap B = \emptyset$$

$$a = n(A), \quad b = n(B)$$

then $n(A) + n(B) = n(A \cup B)$

and therefore $a + b = n(A \cup B)$.

For example, let us consider the two disjoint sets

$$A = \{ a, b, c, d \}$$

and $$B = \{ m, n, p \}$$

and let the union of sets A and B be called set C

$$\therefore \qquad C = \{ a, b, c, d, m, n, p \}.$$

We observe that

$$\begin{aligned} A \cap B &= \varnothing \\ n(A) &= 4 \\ n(B) &= 3 \\ \text{and} \quad n(A \cup B) &= 7 \end{aligned}$$

$\therefore$ by definition

$$\begin{aligned} 4 + 3 &= n(A) + n(B) \\ &= n(A \cup B) \\ &= 7 \end{aligned}$$

Thus the number associated with the union set C is the sum of the numbers associated with the component sets A and B.

So we combine two disjoint sets to add two whole numbers, the cardinal number of their union set is the sum of the cardinal numbers of the two sets. In symbols,

$$n(A \cup B) = n(A) + n(B)$$

iff. $$A \cap B = \varnothing.$$

The two numbers added are called *addends*. Note that we *write* the two sets, and *add* their cardinal numbers.

Now we consider the union of two disjoint sets whose cardinal numbers are less than or equal to 9, and obtain the addition table written below :

Second Number

First number

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

You find in the table, 10 rows and 10 columns. To find

$$8 + 7,$$

take the row against the first number 8 and the column against the second number 7. See where the row and the column meet. They meet at 15. This gives the addition of 8 and 7 as

$$8 + 7 = 15.$$

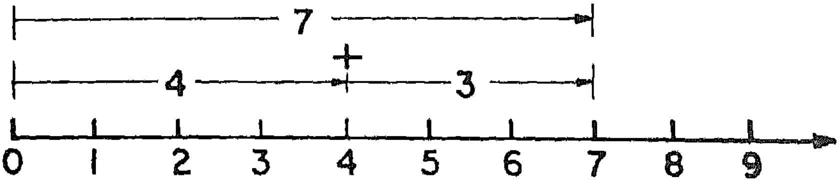
We accomplish addition of large numbers by expressing them in the decimal notation and making use of this addition table.

Addition on the Number-ray

We can illustrate addition on the number-ray as shown below :

To display

$$4 + 3 = 7,$$



start from 0, move a distance of 4 units to reach at 4; then move again a distance of 3 units from 4. You are now at 7, showing that

$$4 + 3 = 7.$$

EXERCISE SET 2-5

Oral

1 Explain the following in terms of addition of numbers associated with the given sets :

(i) $\{g, h, k\} \cup \{p, q, m, n, t\} = \{g, h, k, p, q, m, n, t\}$.

(ii) $\{5, 7, 8\} \cup \{3, 2, 1\} = \{5, 7, 8, 3, 2, 1\}$

(iii) $\{\quad\} \cup \{3\} = \{3\}$

(iv) $\{p, t\} \cup \{\quad\} = \{p, t\}$.

(v) $\{\quad\} \cup \{\quad\} = \{\quad\}$.

2. Complete the following to get true statements :

(i) $n(\{d, k, g\}) + n(\{p, l, t, u, s\}) =$.

(ii) $n(\{5, 4\}) + n(\{\quad\}) =$ ——————.

Written

3. Associate the following addition of numbers with union of sets :

   (i) $5 + 7 = 12$. (ii) $3 + 6 = 9$. (iii) $0 + 4 = 4$.

4. Some students in a class know music and some of them know painting; the cardinal number of all the students in the class is 1845, of those that know music is 1000, of those who know painting is also 1000, what is the number of students who know painting as well as music.

*5. Form several finite sets A and B which may or may not be mutually disjoint, verify that the following is always a true statement :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

*6. Verify for some finite sets A, B, C that you can think of, the following true statement

$$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$

$$- n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

Write the corresponding Venn diagrams.

2-6 Properties through Patterns

Commutative Property

Making use of the addition table, we find certain patterns :

$$5 + 7 = 12 \text{ and } 7 + 5 = 12$$

$$8 + 6 = 14 \text{ and } 6 + 8 = 14$$

and so on.

So we can write

$$5 + 7 = 7 + 5$$

$$8 + 6 = 6 + 8$$

and so on.

Take

$$\square + \triangle = \triangle + \square$$

and see if it fails for *any* two elements of the set of whole numbers :

$$W = \{ 0, 1, 2, 3, \quad \}.$$

You find that it is true for every two whole numbers you choose. This is an important property of whole numbers. It is called the *Commutative property of addition. It tells us :*

In finding the sum of any two whole numbers, they can be taken in any order.

We can not verify this for all pairs of whole numbers, as they form an infinite set. *So we shall accept that for every pair of whole numbers a and b,*

$$a + b = b + a.$$

Associative Property

Suppose you are asked to add 5, 3 and 7. Can you add them all at once ? It is interesting to realise that only two numbers can be added at a time. That is to say,

addition is a two-member operation.

We can add 5 and 3, get 8 and add 8 to 7 and get 15. So we say that 15 is the sum of 5, 3 and 7.

We can also add 3 and 7, get 10 and add 5 and 10 to get 15. So we get the same sum again.

To show that we are adding 5 and 3 first, or 3 and 7 first, we can enclose them in *parentheses* as shown below :

$$(5 + 3) + 7 \quad \text{and} \quad 5 + (3 + 7).$$

Since they give the same sum either way, we write

$$(5 + 3) + 7 = 5 + (3 + 7).$$

We find 3 is *associated with* 5 in the first case and with 7 in the second case. Therefore we write $5+3+7$ for $(5+3)+7$ as well as for $5+(3+7)$.

Take

$$(\square + \triangle) + \bigcirc = \square + (\triangle + \bigcirc)$$

and see if it fails for *any* three whole numbers.

You find that it always gives a true statement for every three whole numbers you choose; this is another important property of whole numbers. It is called *the associative property of addition.* It tells us :

In finding the sum of any three whole numbers, the second can be added to the first and the sum got added to the third or the second can be added to the third and the sum got added to the first.

We cannot verify this property for every set of three whole numbers for the obvious reasons, *so we shall accept it as true. Thus for every three whole numbers a, b and c,*

$$(a+b) + c = a + (b+c).$$

Additive Identity

Consider the addition table as given on page 48. You find an interesting pattern :

$$0 + 0 = 0,$$
$$1 + 0 = 1,$$
$$2 + 0 = 2,$$

and so on.

Also

$$0 + 1 = 1,$$

$$0 + 2 = 2,$$

$$0 + 3 = 3,$$

and so on.

From the above, we find that for every whole number a,

$$a + 0 = a,$$

$$\text{and} \quad 0 + a = a.$$

Therefore 0 is called the *additive identity.*

Closure Property

Take any two whole numbers. Find their sum. Is the sum a whole number ?

You will find that the sum of any two whole numbers is always a whole number, i.e., if a and b be two elements of the set

$$W = \{0, 1, 2, \ldots\},$$

$$\text{then} \qquad a + b \in W.$$

This is called the *closure property of the set of whole number with respect to addition.*

Order Property

Take any two whole numbers such that one is greater than the other, say 9 and 5

$$9 > 5.$$

Is there a whole number which when added to 5 gives 9 ?

You know $5 + 4 = 9$.

Let us try some more examples :

$$11 > 8 \quad ; \quad 8 + 3 = 11$$

$$7 > 6 \quad ; \quad 6 + 1 = 7$$

$$5 > 0 \quad ; \quad 0 + 5 = 5$$

and so on.

This property connects order in the set of whole numbers with addition. We can easily verify :

For any two whole numbers a and b such that a b, there is a whole number c such that

$$a = b + c.$$

By making use of the above property, we can get a better idea of the successor of a whole number.

You know that 7 is the successor of 6 and

$$6 + 1 = 7.$$

Again 1 is the successor of 0 and

$$0 + 1 = 1.$$

So the successor of a whole number is obtained by adding 1 to the number.

Let us take two whole numbers such that one is greater than the other, say, 11 and 6.

Now $$11 > 6.$$

Put *any* whole number in the frames in the following and see if ever you get a false statement

$$11 + \square > 6 + \square.$$

Again take $8 > 1$ and examine

$$8 + \square > 1 + \square,$$

by putting *any* whole number in the frames.

So we shall accept that *for any two whole numbers* a *and* b *such that* $a > b$,

$$a + x > b + x$$

where x *is any whole number.*

EXERCISE SET 2-6

Oral

1 Give two examples for each of the following general statements : (the letters refer to whole numbers)

(i) $a + b = b + a$.
(ii) $(a + b) + c = a + (b + c)$.
(iii) $(a + b) + c = c + (a + b)$.
(iv) If $a > b$, then $a + x > b + x$.
(v) If $a > b$, there is an x such that $a = b + x$.
(vi) $a + 0 = a$.
(vii) $0 + b = b$.

2. Name the property by which each of the following statements are true :

(i) $7 + 3 = 3 + 7$.
(ii) $0 + 8 = 8$.
(iii) $8 + (5 + 6) = (8 + 5) + 6$.

Written

3. Find the set of whole numbers which will make each of the following statements true :

(i) $12 > 3 + \square$.
(ii) $6 > 5 + \square$.

(iii) $6 + \square > 5$.

(iv) $8 = 7 + \square$

(v) $13 = \square + 13$.

(vi) $8 < \square + 3$.

(vii) $15 > 4 + \square$.

2-7 Addition Process

You have learnt to add in earlier classes any number of whole numbers. We shall see what properties and principles are used in the various steps while adding whole numbers having two or more digits. You will learn that the special computational procedures use the principles and properties of numbers, explained in the earlier pages.

Suppose we take the three digit number 362.

It is obviously equal to $300 + 60 + 2$.

Similarly $7148 = 7000 + 100 + 40 + 8$.

Now let us find $638 + 279$.

Writing each of the addends in expanded notation, we have :

$$\begin{aligned} 638 + 279 &= (600 + 30 + 8) + (200 + 70 + 9) \\ &= 600 + 30 + 8 + 200 + 70 + 9 \\ &= (600 + 200) + (30 + 70) + (8 + 9) \\ &= 800 + 100 + 17 \\ &= 900 + 10 + 7 \\ &= 917 \text{ in decimal notation.} \end{aligned}$$

For computational purpose, we can rewrite $638 + 279$ in the vertical form and do the addition in the usual way learnt in the earlier classes.

2-23 Subtraction in W

We shall see under what circumstances subtraction is associated with sets.

Consider a set, say,

$$\{ a, b, c, d, m \}.$$

Remove a subset, say,

$$\{ a, b, c \}.$$

What are you left with ?

The subset $\{ d, m \}$ remains.

Now we can write

$$\text{n} (\{a, b, c, d, m\}) - \text{n} (\{a, b, c\}) = \text{n} (\{d, m\})$$

This is

$$5 - 3 = 2.$$

Combining the two subsets $\{d, m\}$ and $\{a, b, c\}$, we get the original set. That is

$$\{d, m\} \cup \{a, b, c\} = \{a, b, c, d, m\}.$$

So subtraction is related to addition.

The subtraction

$$5 - 3 = 2.$$

is related to the addition

$$2 + 3 = 5.$$

Not only that, since the removal of the subset $\{d, m\}$ from the set $\{a, b, c, d, m\}$ gives the subset $\{a, b, c\}$,

$$\text{n} (\{a, b, c, d, m\}) - \text{n} (\{d, m\}) = \text{n} (\{a, b, c\}),$$

that is

$$5 - 2 = 3.$$

Thus we can see that *to every addition there are two subtractions.*

Examples : (i) Given $6 + 3 = 9$, the two subtractions are :

$$9 - 3 = 6 \quad \text{and} \quad 9 - 6 = 3.$$

(ii) Given $8 + 5 = 13$, the two subtractions are :

$$13 - 8 = 5 \quad \text{and} \quad 13 - 5 = 8.$$

Given one subtraction we can also give the other subtraction in the following way :

Examples : (i) Given $10 - 8 = 2$, the other subtraction is :

$$10 - 2 = 8.$$

(ii) Given $11 - 4 = 7$, the other subtraction is :

$$11 - 7 = 4.$$

Now consider the two sets :

$$A = \{ x, y, z, t, s, u \}$$
$$B = \{ a, b, d, e \}.$$

By setting up one-to-one correspondence between the elements of B and the elements of A in any of the different ways, two elements of A remain unmatchad. See the illustrations given below :

(i)

$$\begin{tikzcd}[column sep=tiny] \{ & x, \arrow[d, leftrightarrow] & y, \arrow[d, leftrightarrow] & z, \arrow[d, leftrightarrow] & t, \arrow[d, leftrightarrow] & s, & u \} \\ \{ & a, & b, & d, & e \} & & \end{tikzcd}$$

The elements of the subset $\{ s, u \}$ are left unmatched.

(ii)

$$\begin{tikzcd}[column sep=tiny] \{ & x, & y, \arrow[d, leftrightarrow] & z, \arrow[d, leftrightarrow] & t, \arrow[d, leftrightarrow] & s, & u \} \arrow[d, leftrightarrow] \\ \{ & & a, & b, & d, & & e \} \end{tikzcd}$$

The elements of the subset $\{ x, s \}$ are left unmatched and so on.

That is to say, the set A has two more elements than the set B.

$$n(A) - n(B) = 2$$

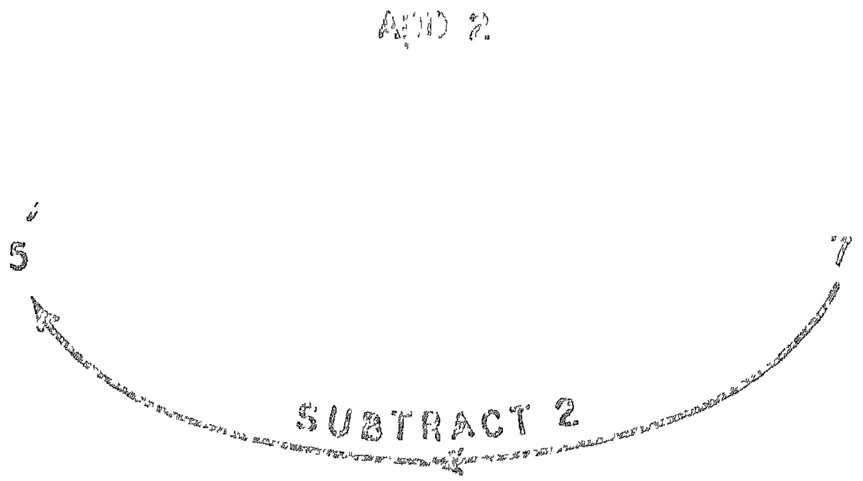
or

$$6 - 4 = 2.$$

We shall take one more situation for considering subtraction.

Suppose you want a set of 9 pencils and you have with you, say, only a set of 4 pencils. A set of 5 pencils combined with your set of 4 pencils gives you the set of 9 pencils. Here the set of 4 pencils is compared with the set of 9 pencils. The set of pencils to be combined is then found out. So we have 9 − 4 = 5.

Subtraction and addition are considered to be inverse operations in a certain sense; see the illustration given below :



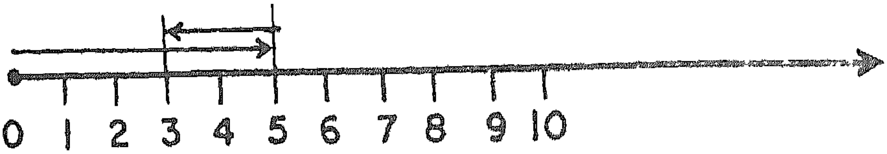
Thus what addition does, is reversed by subtraction.

Subtraction on the Number-ray

We can illustrate subtraction on the number-ray as shown below :

To display

$$5 - 2 = 3,$$



start from 0, move *forward 5 units* and then from 5, move *backward 2 units*. You are at 3 showing

$$5 - 2 = 3.$$

Note : (i) If there are number of additions and subtractions and parentheses are not given, by convention the simplification is done from left to right, generally.

(ii) Consider the set of whole numbers

$$W = \{ 0, 1, 2, 3, \ .. \}$$

In the case of addition, *any* whole number can be added to *any other* whole number to get a whole number in the set, as you have seen earlier; however in the case of subtraction we find that (at the present stage) we can subtract a whole number *a* from *b* and get a whole number iff. $a \leqslant b$, i.e., subtraction is not universally defined in W.

Oral

1. Compare the following pairs of sets and give the corresponding subtractions :

   (i) $\{p, q, l, g, m, v, t, s\}$ and $\{a, b, c, d\}$

   (ii) $\{5, 8, 3\}$ and $\{7, 1, 0\}$

   (iii) { △, ▭, ○, ◇ } and $\{\alpha, \beta, \gamma, \delta, \theta\}$.

2. Give the two subtractions related to each of the following additions :

   (i) $5+6=11$ (ii) $5+7=12$ (iii) $6+8=14$

   (iv) $6+9=15$ (v) $9+9=18$ (vi) $4+0=4$.

3. Give the addition corresponding to each of the subtractions given below :

   (i) $11-5=6$ (ii) $8-8=0$ (iii) $17-9=8$.

Written

4. Write down any 5 subsets of the set $\{p, t, k, m, r\}$ and give the subtractions, when each subset is removed from the set.

5. Interpret $17-11=6$ in as many different ways as you can.

6. Display the following subtractions on the number ray :

   (i) $8-5=3$ (ii) $7-7=0$ (iii) $11-2=9$.

7. Give two subsets of the set $\{a, b, c, l, g, t\}$ such that the intersection of the subsets is

   (i) empty (ii) one element set and (iii) two element set.

8. Find the following :

   (i) $11-2-5$

(ii) $14 - 1 - 6$

(iii) $16 - 4 - 1$

(iv) $(16 - 7) - 3$

(v) $16 - (4 - 1)$

(vi) $8 - 8 - 0$.

9. What whole numbers can be put in the frames in the following to get true statements in each case ?

(i) $8 - \square > 6$ (ii) $10 - \square > 6$ (iii) $5 - \square > 0$

(iv) $7 - \square < 5$ (v) $9 - \square < 10$.

10. Whether the following are whole numbers :

(i) $(16 - 8) - 6$ (ii) $(15 - 7) - 9$ (iii) $(11 - 2) - 4$

(iv) $(10 - 6) - 8$ (v) $12 - (7 - 2)$.

11. What replacements of x from W will make

$$5 - x < 9 - x$$

true ?

12. Find the following :

(i) $18 - 7 - 4 - 2$ (ii) $23 - 8 - 9 + 2 - 4$

(iii) $12 + 2 - 4$ (iv) $13 - 6 + 1$.

13. Verify if the following are true :

(i) $11 - 7 - 2 = 11 - 9$ (ii) $8 + 6 + 2 = 8 + 8$

(iii) $11 - 3 - 3 = 11 - 6$ (iv) $8 + 3 + 4 = 8 + 7$

(v) $11 - 2 - 5 = 11 - 7$ (vi) $8 + 1 + 1 = 8 + 2$.

2-9 Subtraction Procedure involving Large Numbers

When subtraction involves numbers with two or more digits, writing them in *expanded notation* is helpful for understanding the steps in the subtraction process.

As in addition, the vertical form is convenient in doing subtraction.

Study the following examples :

Example 1 : (*Expanded notation*)

$$
\begin{array}{rcrcr}
71 & = & (70 + 1) & = & (60 + 11) \\
-38 & = & -(30 + 8) & = & -(30 + 8) \\
\hline
 & & & & 30 + 3 = 33 \\
\hline
\end{array}
$$

Why is 70 + 1 taken as 60 + 11 ?

8 cannot be subtracted from 1, but 71 > 38, therefore 7 tens and 1 is regrouped as 6 tens and 11 ones so that 8 can be subtracted from 11.

Example 2 : (*Expanded notation*)

$$
\begin{array}{rcrcr}
403 & = & (400 + 3) & = & (300 + 100 + 3) \\
-196 & = & -(100 + 90 + 6) & = & -(100 + 90 + 6) \\
 & & \text{Now} \quad 403 & = & (300 + 90 + 13) \\
 & & \text{and} \quad -196 & = & -(100 + 90 + 6) \\
\hline
 & & & = & 200 + 0 + 7 \\
 & & & = & 207.
\end{array}
$$

When this is done mentally and the expanded notation is not used, you get the familiar form of subtraction procedure.

Oral

1. In which of the following subtractions is renaming (borrowing) necessary ?

(i) 22 — 6 (ii) 37 — 9 (iii) 147 — 51 (iv) 106 — 42

(v) 203 — 67 (vi) 311 — 102 (vii) 100 — 8.

Written

2. Explain the subtraction processes in full ·

(i) $\begin{array}{r} 4376 \\ -\ 829 \\ \hline \end{array}$ (ii) $\begin{array}{r} 6001 \\ -\ 1988 \\ \hline \end{array}$ (iii) $\begin{array}{r} 5222 \\ -\ 4333 \\ \end{array}$

2-10 Application of Subtraction

Subtraction like addition finds application in our daily transactions. While solving story problems, it is helpful to translate them into mathematical sentences. Let us consider a few story problems.

Example 1 : A sewing machine costs Rs. 180. Asha has Rs. 67 with her. How much more does she need to buy the sewing machine ?

Translating it into mathematical sentence,

$$67 + \square = 180.$$

By subtracting 67 from 180, we get 113.

So she needs Rs. 113 more.

Example 2 : A factory employed 320 workers. Some of them did not at all turn up for work and it was found that there were only 208 workers on duty. How many did not turn up for duty ?

Translating it into mathematical sentence,

$$320 - \Diamond = 208.$$

By subtracting 208 from 320, we get 112.

So 112 persons did not turn up for duty.

EXERCISE SET 2-10

Oral

1. Translate the following story problems into mathematical sentences using (×) instead of number frame. Give the answers also.

   (i) John bought for his son Richard a dozen note-books. Richard lost 3 of them a few days after his school reopened. How many note-books are left with him ?

   (ii) A tin contained 10 litres of petrol. After a week only 9 litres of petrol remained in it. How much petrol evaporated during the week ?

   (iii) In a class during a year names of 8 pupils were struck off the rolls. If at the end of the year 35 pupils were on rolls, what was the strength of the class at the begining of the year ?

Written

2. Complete the following suitably :

   He has 8 mangoes with him. He needs 12 mangoes.

   How many more____________ does he need ?

   He needs______________mangoes.

3. Ask a question to make the following a subtraction problem ·

   (i) Kesav is 150 cm tall. His younger brother Vikram is 160 cm tall.

   (ii) My father wants to buy a house for Rs. 25,000. He has a savings of Rs. 16,750.

4. The population of a town in 1950 was 17,860. It increased by 4,000 in the next ten years. In 1961 an earthquake occurred and the population was reduced to 12,590. How many died during the earthquake ?

2-11 Multiplication in W

We have seen that by finding the union of disjoint sets we discover the addition of numbers.

We shall now find the union of equivalent disjoint sets.

Consider three equivalent disjoint sets with 2 elements in each, i.e., *with the same cardinal number 2.*

Suppose they are $\{ a, b \}$, $\{ d, e \}$ and $\{ l, t \}$.

How will you find their union ?

$$\{ a, b \} \cup \{ d, e \} = \{ a, b, d, e \}$$

$$\begin{aligned} \{ a, b \} \cup \{ d, e \} \cup \{ l, t \} &= \{ a, b, d, e \} \cup \{ l, t \} \\ &= \{ a, b, d, e, l, t \}. \end{aligned}$$

(You could have taken the union of any other two of the above sets first and then found the union of the so obtained union set with the third remaining set)

How many elements are there in the set $\{a, b, d, e, l, t\}$? There are six elements.

Now $$n (\{ a, b \}) + n (\{ d, e \}) + n(\{ l, t \}) = n (\{ a, b, d, e, l, t \}).$$

That is $$2 + 2 + 2 = 6.$$

Counting the two's, we find three two's are equal to six. It is written in the short form as

$$3 \times 2 = 6,$$

which gives the *multiplication of numbers*

Let us consider two equivalent disjoint sets with 5 elements in each.

$$\{ a, k, l, t, u \} \cup \{ b, c, d, e, g \} = \{ a, k, l, t, u, b, c, d, e, g \}.$$

So $$n (\{ a, k, l, t, u \}) + n (\{ b, c, d, e, g \})$$
$$= n (\{ a, k, l, t, u, b, c, d, e, g \})$$

That is $$5 + 5 = 10.$$

Also counting the five's, we find two five's are equal to 10, i.e.,

$$2 \times 5 = 10.$$

Thus multiplication of any two whole numbers is associated with the union of equivalent disjoint sets.

Two numbers multiplied are called *factors* and the result is called the *product*.

Multiplication arises in another interesting situation. Consider the following problem :

A set of three boys

{ *Ram, Lal, Kumar* }

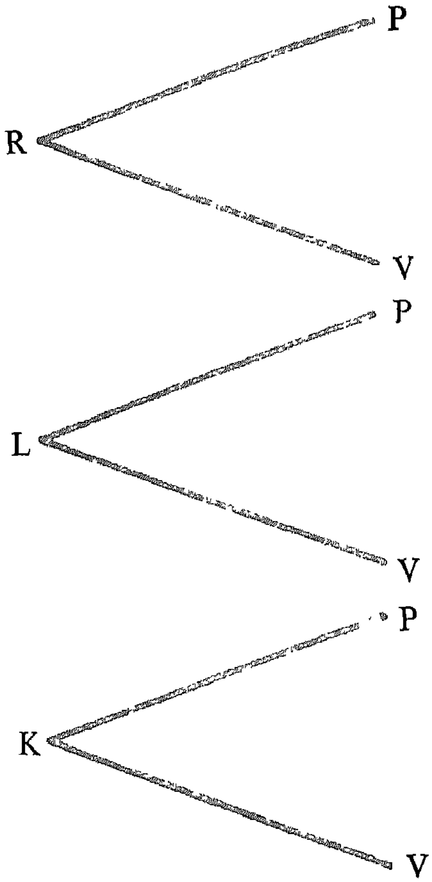
want to take group photos each with each one of their favourite teachers

{ *Premnath, Vivekanand* }.

How many photos have to be taken ?

The answer to this question can be obtained from the fact that each student shall have two photos, one with Premnath and the other with Vivekanand. These pairings can be exhibited by a tree-diagram as shown below. The boys and teachers are denoted by the first letter of their names. Thus 'P' stands for Premnath, etc.,

Tree diagram



This question can be tackled in another way also. In each photo, there shall be one teacher and one student, and the total number of such different pairings will give the total number of photos to be taken. How many such pairings are possible, can be found by tabulating them as shown below in the tabular form :

Tabular form

(i)

	R	L	K
P	(P, R)	(P, L)	(P, K)
V	(V, R)	(V, L)	(V, K)

or

(ii)

	P	V
R	(R, P)	(R, V)
L	(L, P)	(L, V)
K	(K, P)	(K, V)

This can be written as

{ (R, P), (R, V), (L, P), (L, V), (K, P), (K, V) }.

A set of all such possible pairings as

{ (R, P), (R, V), (L, P), (L, V), (K, P), (K, V) },

where the first element of each pairing is of the set { R, L, K } and the second element is from the set { P, V } is called the *cartesian product* of these two sets or simply the product set denoted by

{ R, L, K } × { P, V }.

Also

The cardinal number of the cartesian product of the two sets is the product of the cardinal numbers of the two sets.

Rectangular Array

An arrangement, consisting of elements, placed in horizontal rows and vertical columns is called a *rectangular array.* For example the arrangement

$$\begin{array}{ccc} a & b & c \\ e & f & g \end{array}$$

is called a rectangular array of 2 by 3 elements as there are two horizontal rows and 3 vertical columns.

Also, the number of elements in a 2 by 3 rectangular array, arranged in two rows and three columns as above, gives the cartesian product of that array. The following rectangular arrays represent the same product 2 × 3, which is equal to 6 :

□ □ □

□ □ □

or

× × ×

× × ×

or

○ ○ ○

○ ○ ○

When we have 1 by 2 array, we get the multiplication

$$1 \times 2 = 2.$$

Similarly we can find 1×3, 1×4, etc. If we take 2 by 1 array, we get the multiplication as

$$2 \times 1 = 2.$$

Similarly we can find 3×1, 4×1, etc.

Now what is 3×0?

We can find it by studying the pattern in the following :

$$3 \times 4 = 12$$

$$3 \times 3 = 9$$

$$3 \times 2 = 6$$

$$3 \times 1 = 3$$

$$3 \times 0 = \square$$

Products decrease by 3 at each step. So

$$3 \times 0 = 0.$$

Similarly we can find 4×0, 5×0, etc.

Now what is 0×4?

Again we shall find it by studying the pattern in the following :

$$5 \times 4 = 20$$

$$4 \times 4 = 16$$

$$3 \times 4 = 12$$

$$2 \times 4 = 8$$

$$1 \times 4 = 4$$

$$0 \times 4 = \square$$

Products decrease by 4 at each step. So

$$0 \times 4 = 0.$$

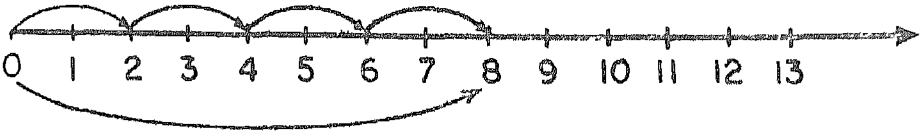
The following table called the '*Multiplication Table*' gives the product of any two of the one digit numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Second Number

	×	0	1	2	3	4	5	6	7	8	9
First Number	0	0	0	0	0	0	0	0	0	0	0
	1	0	1	2	3	4	5	6	7	8	9
	2	0	2	4	6	8	10	12	14	16	18
	3	0	3	6	9	12	15	18	21	24	27
	4	0	4	8	12	16	20	24	28	32	36
	5	0	5	10	15	20	25	30	35	40	45
	6	0	6	12	18	24	30	36	42	48	54
	7	0	7	14	21	28	35	42	49	56	63
	8	0	8	16	24	32	40	48	56	64	72
	9	0	9	18	27	36	45	54	63	72	81

Multiplication on the number-ray

Multiplication can also be shown on the number-ray.



The diagram represents the multiplication of numbers 4 and 2 as

$$4 \times 2 = 8.$$

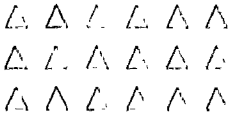
It is to be interpreted as 4 'jumps' of 2 units each.

EXERCISE SET 2·11

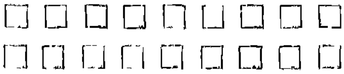
Oral

1. Give the multiplication for each of the arrays given below :

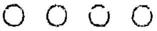
(i)



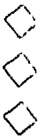
(ii)



(iii)

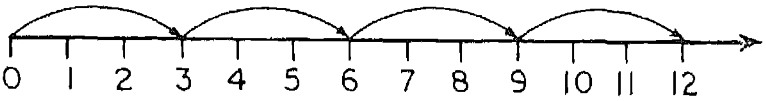


(iv)

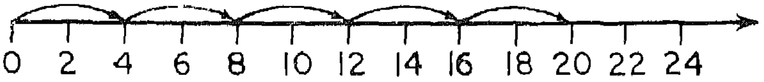


3. Give the multiplication represented by each of the number-ray diagrams given below :

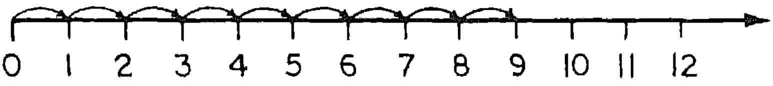
(i)



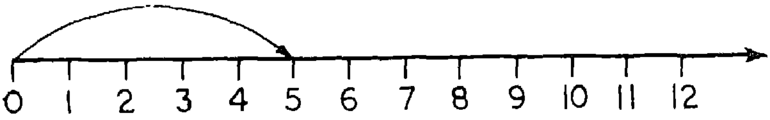
(ii)



(iii)

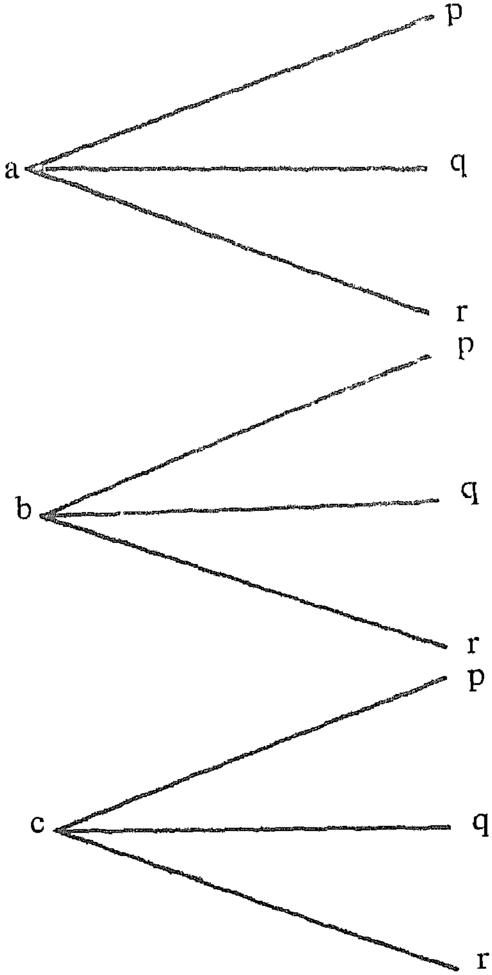


(iv)

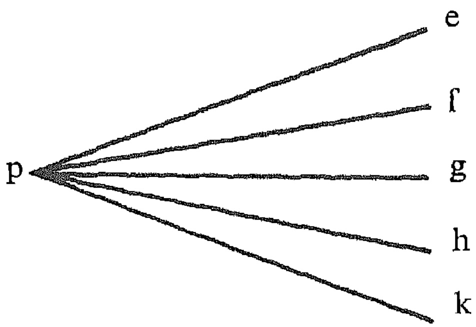


4. Give the multiplication represented by each of the three-diagrams given below :

(i)



(ii)



4. Write the following in multiplication form .

(i) $7 + 7 + 7 = 21$

(ii) $5 + 5 + 5 + 5 + 5 + 5 = 30$.

(iii) $0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$.

(iv) $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8$.

(v) $2 + 2 + 2 + 2 = 8$.

(vi) $10 + 10 + 10 = 30$.

(vii) $100 + 100 + 100 + 100 = 400$

(viii) $1000 + 1000 = 2000$.

(ix) $10+10+10+10+10+10+10+10+10+10=100$.

(x) $100+100+100+100+100+100=600$.

Written

5. Draw an array of crosses (×) for each of the following multiplications .

(i) $8 \times 4 = 32$. (ii) $4 \times 8 = 32$. (iii) $6 \times 1 = 6$

(iv) $1 \times 6 = 6$. (v) $5 \times 5 = 25$.

6. Use a number-ray to depict the following products :

(i) 5×3. (ii) 3×3. (iii) 8×1 (iv) 1×9.

7. Draw a tree-diagram to represent each of the following products :

(i) 3×6. (ii) 6×3. (iii) 5×1. (iv) 1×5.

8. Complete the following and continue for next two stages :

(i) $3 \times 10 = \square$ (ii) $6 \times 100 = \square$ (iii) $5 \times 1000 = \square$

$4 \times 10 = \square$ $9 \times 100 = \square$ $7 \times 1000 = \square$.

9. Find all the number-pairs to fill the two frames to get true statements:

(i) □ × △ = 12. (ii) □ × △ = 15.

10. At breakfast time a hostel offers the choice of dosa, iddli or puri for solid food and for drink, the choice is coffee or tea. Give in the form of a table all the combinations of a solid and a drink.

2-12 Properties of Multiplication through Patterns

Commutative property

As in addition, commutative property of multiplication relates to two numbers.

From the multiplication table on page 72, we find that

$$3 \times 5 = 5 \times 3$$

$$4 \times 7 = 7 \times 4$$

$$8 \times 1 = 1 \times 8$$

$$6 \times 0 = 0 \times 6$$

$$9 \times 2 = 2 \times 9.$$

Find out by filling in any numbers for a and b in

$$a \times b = b \times a,$$

whether you ever get an example of a false statement.

It comes out true for every pair of whole numbers you choose.

As one cannot verify for all pairs of whole numbers, we shall assume that in multiplying any two whole numbers, their product is the same whatever be their order.

This is called the *commutative property* of multiplication in the set of whole numbers. It can be stated as follows :

For every two whole numbers a and b,

$$a \times b = b \times a.$$

Note : While using letters, multiplication sign can be dropped and $a \times b = b \times a$ can simply be written as $ab = ba$.

Associative property

As in addition, associative property of multiplication relates to three numbers.

Now what is $3 \times 2 \times 7$?

As multiplication is a *two-number operation*, we multiply 3 and 2 first, get 6 and then multiply 6 by 7. We get the product 42.

This can be shown by using *parentheses*. Numbers inside the parentheses should be multiplied first.

$$(3 \times 2) \times 7 = 42$$

Also $$3 \times (2 \times 7) = 42.$$

So we have

$$(3 \times 2) \times 7 = 3 \times (2 \times 7).$$

This shows that 2 can be associated with 3 or 7 first and the product got multiplied by the remaining third number.

Find by filling in the frames

$$(\triangle \times \square) \times \Diamond = \triangle \times (\square \times \Diamond),$$

whether you get an example of a false statement. It comes out true for every three whole numbers chosen.

This property, called the *associate property* of the set of whole numbers with respect to multiplication, can be stated as follows :

For every three whole numbers a, b, c,

$$(a \times b) \times c = a \times (b \times c).$$

Note · Omitting the multiplication signs as before,

$$(a \times b) \times c = a \times (b \times c)$$

can simply be written as

$$(ab)c = a(bc).$$

A combination of commutative and associative properties is highly useful in multiplication computation.

Study the following examples :

$(3 \times 2) \times 5 = (2 \times 3) \times 5$ (commutative property)

$= 2 \times (3 \times 5)$ (Associative property)

$= 2 \times (5 \times 3)$ (commutative property)

$\therefore \quad (3 \times 2) \times 5 = 2 \times (5 \times 3).$

This shows that in multiplication of three whole numbers, the numbers can be *rearranged* in any order and the product remains unchanged.

Let us use these properties in finding products in the following examples.

Example : 1. Find $20 \times 30 \times 5$.

Solution : $20 \times 30 \times 5 = 2 \times 10 \times 3 \times 10 \times 5$

$= (2 \times 3) \times 5 \times (10 \times 10)$

$= 6 \times 5 \times 100$

$= 3000$

Example : 2. Find $2 \times 4 \times 5 \times 3 \times 5$.

Solution :

$$\begin{aligned} 2 \times 4 \times 5 \times 3 \times 5 &= 3 \times (4 \times 5) \times (2 \times 5) \\ &= 3 \times 20 \times 10 \\ &= (3 \times 2) \times (10 \times 10) \\ &= 6 \times 100 = 600. \end{aligned}$$

Distributive Property

Besides commutative and associative properties, there is another very important property. We shall discover it by considering a few problems.

Example : 1. A boy undertook to sell Rs. 5 tickets in aid of school building fund. He sold on Monday 3 tickets and on Tuesday 4 tickets. How much money should be with him at the end of Tuesday ?

The problem can be solved in two ways :

(i) Cash with him by the sale of tickets on Monday is

$$\text{Rs. } 3 \times 5 = \text{Rs. } 15.$$

Cash with him by the sale of tickets on Tuesday is

$$\text{Rs. } 4 \times 5 = \text{Rs. } 20.$$

Total Cash with him at the end of Tuesday is

$$\text{Rs. } (15 + 20) = \text{Rs } 35.$$

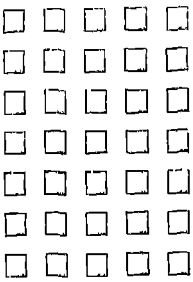
Or

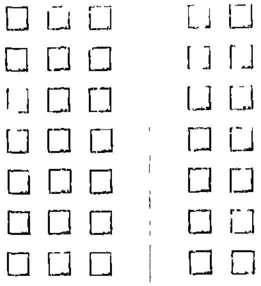
(ii) Total number of Rs. 5 tickets sold by Tuesday $(3+4) = 7$.
Total cash with him at the end of Tuesday is

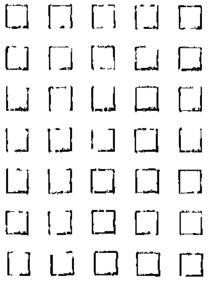
$$\begin{aligned} \text{Rs. } (3+4) \times 5 &= \text{Rs. } 7 \times 5 \\ &= \text{Rs. } 35. \end{aligned}$$

So $\quad (3 + 4) \times 5 = 3 \times 5 + 4 \times 5.$

Example : 2. An array is given below. The same is shown split up in two ways. Write down for each the multiplication expression and then write the true statements.







Solution :

We see that the first array is 7 by 5 array. The next picture is of the same array but the 5 columns have been spilt up into 3 and 2 columns. Thus the split up arrays are now 7 by 3 and 7 by 2 arrays.

Thus we can say that

$$7 \times 5 = 7 \times (3 + 2)$$

$$= (7 \times 3) + (7 \times 2).$$

Similarly in the third picture the 7 rows have been split up into 3 and 4 rows giving rise to two arrays, one 3 by 5 and the other 4 by 5, thus we have

$$7 \times 5 = (3 + 4) \times 5$$

$$= (3 \times 5) + (4 \times 5).$$

Thus the two statements are

(i) $7 \times (3 + 2) = (7 \times 3) + (7 \times 2)$.

(ii) $(3 + 4) \times 5 = (3 \times 5) + (4 \times 5)$.

Now find by filling the frames in

$$\square\,(\triangle + \diamondsuit) = (\square \times \triangle) + (\square \times \diamondsuit)$$

with whole numbers, if a false statement is got. There is no false statement. We shall accept this property called the *distributive property of whole numbers over addition,* as we cannot verify it for all whole numbers. It can be stated as :

For all whole numbers a, b, c

$$a \times (b + c) = (a \times b) + (a \times c)$$

and $$(b + c) \times a = (b \times a) + (c \times a)$$

Note : Omitting the multiplication sign we can write these simply as :

$$a\,(b + c) = ab + ac$$

and $$(b + c)\,a = ba + ca.$$

Multiplicative Identity

For any whole number a, the following is true

$$1 \times a = a,$$

and $$a \times 1 = a.$$

Therefore, 1 is named the *multiplicative identity.*

Thus, what 0 does when it is added to a whole number, 1 does when it multiplies a whole number.

Multiplicative property of zero

When a whole number is multiplied by zero, the product is always zero.

Closure Property

We see that the product of two whole numbers is a whole number. That is to say,

For any two elements a, b of the set of whole numbers

$$W = \{ 0, 1, 2, 3, \dots \},$$

the product $ab \in W$.

We therefore say that the set W is closed with respect to multiplication.

Order and Multiplication

Take any two whole numbers. If they are not the same, you know that one is greater than the other. Let them be 7 and 4.

Now $7 > 4.$

What whole numbers can be put in the frame

$$7 \times \square > 4 \times \square$$

to get true statements ?

When 0 is put, we get

$$7 \times 0 > 4 \times 0,$$

which is a false statement.

For any other whole number,

$$7 \times \square > 4 \times \square$$

gives a true statement. Thus :

The order of two whole numbers remains unchanged if they are multiplied by any non-zero whole number (or by any natural number).

EXERCISE SET 2-12

Oral

1. State the properties used in each of the following :

(i) $2 \times 7 \times 3 = 2 \times 3 \times 7.$

(ii) $5 \times 6 \times 4 = 6 \times 4 \times 5.$

(iii) $(20 + 5) \times 4 = 20 \times 4 + 5 \times 4.$

(iv) $6 \times 2 \times 5 \times 3 = 6 \times 5 \times 2 \times 3.$

2. Find the following by using the distributive property :

(i) $(3 \times 8) + (7 \times 8)$.

(ii) $(2 \times 9) + (3 \times 9) + (5 \times 9)$.

(iii) $(6 \times 7) + 6 \times (2 + 6)$

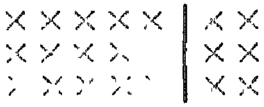
(iv) $5 + 5 \times (8 + 5)$.

3. The product and sum of two whole numbers are given in the following table; find the numbers :

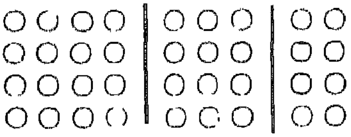
	Product	*Sum*
(i)	12	8
(ii)	20	9
(iii)	8	6
(iv)	7	8
(v)	0	4
(vi)	1	2

4. Give the statement for each of the split arrays shown below :

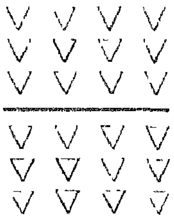
(i)



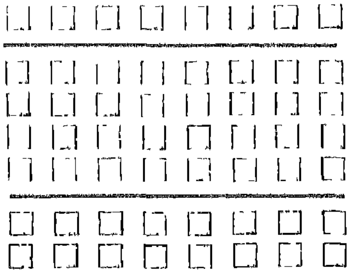
(ii)



(iii)



(iv)



5. Illustrate the following statements by arrays :

(i) $8 \times (5+4+1) = (8 \times 5)+(8 \times 4)+(8 \times 1)$.

(ii) $(3+3+4+2) \times 4 = (3 \times 4)+(3 \times 4)+(4 \times 4)+(2 \times 4)$.

Written

6. Examine if the following statements are true :

(i) $5 \times (8 - 6) = (5 \times 8) - (5 \times 6)$.

(ii) $7 \times (5 - 4) = (7 \times 5) - (7 \times 4)$.

(iii) $(7 - 3) \times 9 = (7 \times 9) - (3 \times 9)$.

(iv) $(9 - 2) \times 6 = (9 \times 6) - (2 \times 6)$.

7. Simplify using the distributive property :

(i) $12 \times 983 + 46 \times 983 + 42 \times 983$.

(ii) $87 \times 649 - 86 \times 649$.

(iii) $374 \times 7894 + 626 \times 7894$.

8. (i) If the product of two non-zero whole numbers is one of the numbers, can you find always the other number ?

(ii) If the product of two whole numbers is zero, what can you say about the numbers ?

9. Find the set of whole numbers which when filled in the frame in each of the following gives true statements :

(i) $\Diamond \times 7 < 58$.

(ii) $\Diamond \times 2 < 40$.

(iii) $69 > \Diamond \times 8$.

2-13 Multiplication (Procedure) for large numbers

With the properties listed in the previous section, the computation procedure (or algorithm) in multiplication of two numbers having two or more than two digits can be easily explained.

For example :

(i) $5 \times 237 = 5 \times (200 + 30 + 7)$

$$\begin{aligned}
&= (5 \times 200) + (5 \times 30) + (5 \times 7) \quad \text{(distributive property)}\\
&= 1000 + 150 + 35\\
&= 1000 + (100 + 50) + (30 + 5)\\
&= 1000 + 100 + (50 + 30) + 5\\
&= 1000 + 100 + 80 + 5\\
&= 1185.
\end{aligned}$$

This example is worked out in the usual vertical form as follows :

$$\begin{array}{rl}
237 & \\
\times\ 5 & \\
\hline
35 & (5 \times 7)\\
150 & (5 \times 30)\\
1000 & (5 \times 200)\\
\hline
1185 & \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
237\\
\times\ 5\\
\hline
1185\\
\hline
\end{array}$$

(ii) $46 \times 353 = (40 + 6) \times 353$

$$\begin{aligned}
&= (40 \times 353) + (6 \times 353)\\
&= 14120 + 2118\\
&= 16238.
\end{aligned}$$

Also $46 \times 353 = 353 \times 46$ (Commutative Property)

and
$$\begin{array}{r} 353 \\ \times\ 46 \\ \hline 2118 \\ 1412 \\ \hline 16238 \end{array}$$

$\therefore \quad 46 \times 353 = 16238.$

Use of brackets

Parentheses, as you have seen, are used to indicate which operation is done first. For example :

(i) $7 \times (5 + 2)$ shows 5 and 2 should be added first and the sum multiplied by 7.

(ii) $(7 \times 5) + 2$ shows 7×5 are multiplied first and the product got is added to 2.

Note : $3 \times (5-2)$ can be simply written as $3\,(5-2)$ and

$(5 - 2) \times (2 + 1)$ as $(5 - 2)\,(2 + 1)$.

EXERCISE SET 2-13

Oral

1. Give two examples for each of the following general statements : (the letters refer to whole numbers)

(i) $a \times b = b \times a$.

(ii) $a \times (b \times c) = (a \times b) \times c$.

(iii) $a \times (b + c) = (a \times b) + (a \times c)$.

(iv) $a \times (b - c) = (a \times b) - (a \times c)$.

(v) $(a + b) \times c = (a \times c) + (b \times c)$.

Written

2. Rewrite the following dropping multiplication sign, wherever possible :

   (i) $3 \times (5 + 6)$. (ii) $8 \times (9 - 1)$. (iii) 28×36.

   (iv) $a \times (p + q) = (a \times p) + (a \times q)$.

   (v) $m \times n = n \times m$. (vi) $(3 + 2) \times (3 - 2)$.

3. Simplify the following :

   (i) $(7 \times 2) + 1$. (ii) $(4 \times 8) - 5$.

   (iii) $(6 \times 3) - (2 \times 2)$. (iv) $(3 \times 5) + (4 \times 2) - (6 \times 1)$.

   (v) $3(7 - 3)$. (vi) $(4 - 2)(3 + 1)$.

5. Insert parentheses to make the following statements true :

   (i) $3 \times 4 + 1 = 15$.

   (ii) $7 \times 2 + 6 = 20$.

   (iii) $2 \times 4 + 3 \times 5 = 23$.

   (iv) $6 \times 5 + 2 \times 3 = 126$.

   (v) $8 \times 5 - 3 = 16$.

6. Find the amount of the bill after filling the blanks :

Quantity	*Item*	*Rate*	*Amount*
10 bars	Bar soaps	Rs. 0.60 each	______
4 kg	Sugar	Rs. 3.70 per kg.	______
4 litres	Kerosene	Rs. 2.10 per litre	
2 bottle	Horlicks	Rs. 6.80 each	______
		Total Rs.	______

7. Find the amount in the cash box after filling the blanks :

Number	Coin	Amount Rs
180	2 paise	
55	3 paise	
241	5 paise	
64	10 paise	
92	25 paise	
	Total Rs.	

2-14 Division in W

We have seen that addition is associated with the union of disjoint sets, subtraction is associated with the removal of a subset from a set, multiplication is associated with the union of equivalent disjoint sets or the cartesion product of two sets.

It therefore seems natural to believe that division may be associated with the partitioning of a set into equivalent disjoint subsets. We shall examine it.

Consider a set, say,

$$A = \{l, t, u, k, d, e, f, m\}.$$

Into what equivalent, disjoint subsets can it be partitioned ? We shall partition it into subsets of one element each.

The subsets are

$$\{l\}, \{t\}, \{u\}, \{k\}, \{d\}, \{e\}, \{f\}, \{m\}.$$

They are 8 in number.

This partition can be expressed as the division

$$8 \div 1 = 8.$$

We can also partition set A into equivalent disjoint subsets of 2 elements each The subsets can be

$$\{l, t\}, \{u, k\}, \{d, e\}, \{f, m\}.$$

They are 4 in number.

This partition can be expressed as the division

$$8 \div 2 = 4.$$

We shall now partition set A into equivalent disjoint subsets of 4 elements each. The subsets can be

$$\{l, t, u, k\}, \qquad \{d, e, f, m,\}$$

They are 2 in number.

This partition can be expressed as

$$8 \div 4 = 2.$$

If set A is partitioned into subsets of 8 elements each, there is only one subset, that is the set itself.

This partition can be expressed as

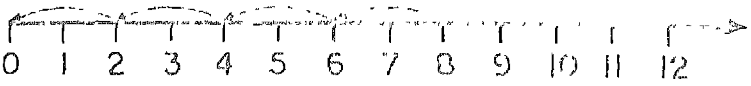
$$8 \div 8 = 1.$$

Thus partitioning of a set into equivalent disjoint sets corresponds to division

Since partition of any set by void set is not possible, division by zero is not admissible.

Division on Number-ray

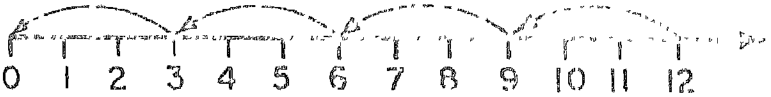
Suppose we want to illustrate the division of 8 by 2 with the help of the number-ray. We can do it as follows :



Starting from 8 we take jumps of 2 units each backwards till we reach at 0. This gives the division

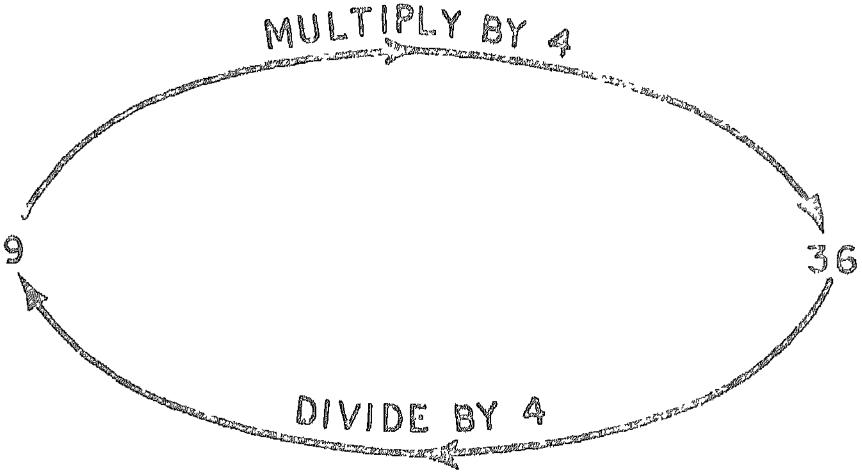
$$8 \div 2 = 4.$$

Similarly $12 \div 3 = 4$ can be depicted on number-ray as follows :



Note that division can also be looked upon as counting backwards. $12 - 3$ is counting backwards from 12 in threes.

Division is related to multiplication as subtraction is related to addition Division is an inverse operation to multiplication in a certain sense. See the diagram below :



Thus what multiplication does, is reversed by division.

Note that corresponding to every multiplication of a and b there are two divisions provided that a and b are not equal to 0, i.e., given the multiplication

$$a \times b = c, \quad a \neq 0, b \neq 0;$$

we have

$$c \div a = b \quad \text{and} \quad c \div b = a.$$

Examples : (i) Given $3 \times 2 = 6$, we can state the two related divisions as

$$6 \div 3 = 2 \quad \text{and} \quad 6 \div 2 = 3.$$

(ii) Given $8 \times 7 = 56$, the two divisions are

$$56 \div 8 = 7 \quad \text{and} \quad 56 \div 7 = 8.$$

To every division there is a related multiplication

Given

$$10 \div 2 = 5,$$

the related multiplication is

$$5 \times 2 = 10.$$

We know that any whole number multiplied by 0 is always 0. So there is no whole number which when multiplied by 0 gives a non-zero whole number. Thus,

Division by zero is meaningless.

In other words, division by zero is not at all defined in W.

Division with remainder

Consider the set

$$A = \{ l, t, u, k, d, e, f, m, n, p, q\}.$$

You try to partition it into equivalent disjoint subsets of 3 elements each, always *two* elements are left out.

In such a case we say that when 11 is divided by 3, the quotient is 3 and the remainder is 2 and it can be written as ·

$$11 = 3 \times 3 + 2$$

or

$$\begin{array}{r} 3 \\ 3\overline{)11} \\ 9 \\ \hline 2 \end{array} \quad \text{Remainder} = 2.$$

The number which is divided is called the *dividend* The number by which the dividend is divided is called the *divisor*. The number resulting from the division of one number by another is called the *quotient*. The difference between the dividend and the *product* of the divisor and the quotient is called the *remainder*. For example consider $13 \div 4$.

Now $\quad 13 = (4 \times 3) + 1.$

Hence 13 is the dividend, 4 the divisor, 3 the quotient and 1 the remainder.

Note : In case the divisor is greater than the dividend, the quotient is zero and the remainder is the dividend itself.

For any two whole numbers a and b, $(a > b)$, examine if a whole number exists to put in the frame in

$$a = b \times \triangle$$

to get a true statement, it may not exist; however you will find that whole numbers always exist to put in the frames in

$$a = b \times \square + \triangle$$

to get true statements. For example, no whole number if put in the frame makes

$$23 = 4 \times \triangle$$

a true statement. In such a case consider

$$23 = 4 \times \square + \triangle.$$

You can find whole numbers which if put in the frames, give a true statement, e.g.,

$$23 = 4 \times 1 + 19$$

$$23 = 4 \times 2 + 15$$

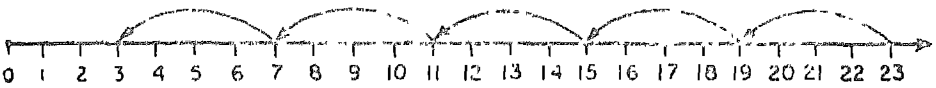
$$23 = 4 \times 3 + 11$$

$$23 = 4 + 4 + 7$$

$$23 = 4 \times 5 + 3$$

You cannot proceed further. You see in the last case that the remainder 3 is less than the divisor. So 23 divided by 4 gives the quotient 5 and the remainder 3.

This can also be shown on number-ray as :



From the above discussion, we find :

For any two whole numbers a and b, (b being a divisor not equal to zero) there always exist two whole numbers q and r such that

$$a = bq + r,$$

where $0 \leq r < b$.

EXERCISE SET 2-14

Oral

1. Give the two divisions related to each of the following multiplications :

(i) $9 \times 8 = 72$. (ii) $7 \times 6 = 42$. (iii) $4 \times 1 = 4$.

(iv) $8 \times 7 = 56$. (v) $10 \times 5 = 50$. (iv) $1 \times 13 = 13$.

2. Give the multiplication related to each of the following divisions :

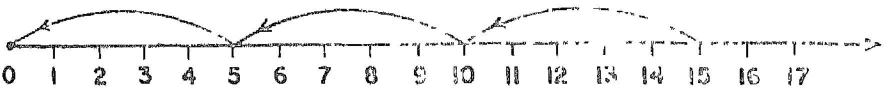
(i) 64÷8 = 8. (ii) 30÷5 = 6. (iii) 0÷8 = 0.

(iv) 28÷7 = 4. (v) 6÷6 = 1. (vi) 9÷1 = 9.

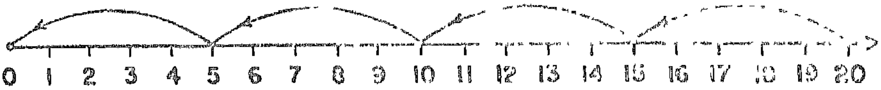
(vii) 22÷2 = 11. (viii) 22÷11 = 2.

3. State the division depicted on number-ray below :

(i)

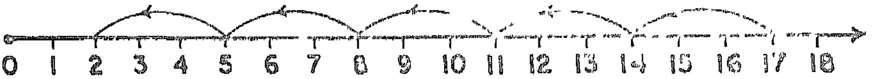


(ii)

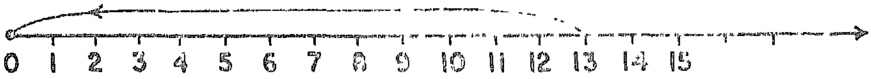


4. Find the remainder of the division depicted on number-ray below :

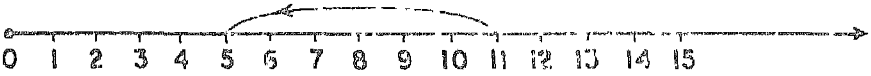
(i)



(ii)



(iii)



5. 6 copies of a book can be placed in a row. How many such rows do I need to place 48 copies of the book ?

6. If 56 meters of cloth is distributed equally among 7 customers, how much does each of them get ?

7. There are two sticks one painted green and the other red. The green one is 8 cm long and the red one 40 cm long. What times the length of the green one is the red one ?

8. Insert × or ÷ in the frames to get true statement :

(i) $105 \square 7 = 15$ (ii) $3 \square 9 = 27$

Written

9. Find whole numbers a and b to get true statements so that the remainder in each be less than the divisor :

(i) $678 = (31 \times a) + b = 31a + b$.

(ii) $4543 = 87a + b$.

(iii) $1729 = 13a + b$.

(iv) $571428 = 142857a + b$.

(v) $8 = 17a + b$

10. Depict the following facts on number-ray :

(i) $3 \times 8 = 24$. (ii) $2 \times 7 = 14$. (iii) $4 \times 7 + 3 = 31$.

*11. Take a whole number and divide it by 8 and you study the remainder in each case. What are the possible remainders ? Do they form a finite or an infinite set ?

2-15 Applications of Division

While applying division, you should be careful in interpreting the significance of the remainder according to the situation.

Example · *1.* 43 guests are to be taken to the railway station. Only 5 guests can be taken in one trip by car. In how many trips can they be taken to the station ?

Solution : 43 divided by 5 gives the quotient 8 and the reminder 3. That means after 8 trips, 3 guests will remain. To take them to the station, another trip is necessary. So in 9 trips, all the guests can be taken to the station.

Example : *2.* 77 competitors are to be seated on long benches. 6 competitors alone can sit on a long bench. How many benches are needed ?

Solution : 77 divided by 6 gives the quotient 12 and the remainder 5.

That means, the number of benches that are needed is

$$12+1 = 13.$$

Example : *3.* A shirt requires 2 meter 30 centimeters of cloth. How many shirts can be made from 9 meter 70 centimeters of cloth ? Does any piece of cloth remain over ?

Solution : (In such examples we shall write the measuremennts in subunits only, i.e., here in centimeters)

Cloth required for one shirt is 2 m 30 cm or 230 cm

Length of cloth available is 9 m 70 cm or 970 cm

$$\begin{array}{r} 4 \\ 230\,\overline{)\,970} \\ 920 \\ \hline 50 \end{array}$$

So, the number of shirts that can be got is 4.

There will remain a cloth piece of 50 cm length.

EXERCISE SET 2-15

Written

1. A bag can hold 2 kg of flour. I want to pack 313 kg of flour in bags. How many bags will I need ?

2 A soap box can hold 400 cakes of soap of a certain kind. Raju collected 9547 such cakes. In how many boxes could he put them ?

3. If 832 banana saplings are planted, 7 in a row, how many more are needed to complete the incomplete row ?

4. A machine turns out a tin once every 2 minutes. In one hour how many tins does it turn out ?

5. Sekhar opened a basket containing 27 mangoes. He removed the rotten ones and found that he could distribute the remaining ones equally among his 6 cousins. If the rotten ones were less than six, find their exact number.

6. If 28 ten-paise coins and 404 five-paise coins are exchanged for twenty five paise coins, find the number of twenty five paise coins.

2-16 Exponents

You have seen that repeated addition of a whole number is indicated in short form as a product. For example,

$$5 + 5 + 5 + 5 = 4 \times 5$$

$$7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 8 \times 7.$$

Similarly, repeated multiplication of a whole number is indicated in short form as a power. It is a product of equal factors.

For example,

$$5 \times 5 \times 5 \times 5 = 5^4$$

$$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^8.$$

5^4 is a power of 5 and is read :

5 raised to the power 4 or *4th power of 5.*

7^8 is a power of 7 and is read :

7 raised to the power 8 or *8th power of 7.*

The numeral placed at the right of and above a numeral is called the exponent (or index). *The number which is raised to a power is called the base.*

In 5^4, 5 is the base and 4 is the exponent.

In 7^8, 7 is the base and 8 is the exponent.

Place values in a decimal system of numerals can now be given as powers of ten :

$$
\begin{aligned}
10 &= 10^1 \\
100 &= 10 \times 10 = 10^2 \\
1000 &= 10 \times 10 \times 10 = 10^3 \\
10000 &= 10 \times 10 \times 10 \times 10 = 10^4
\end{aligned}
$$

and so on.

The pattern shows that the number of zeros in a power of ten is the same as the exponent. A whole number can now be given in exponential form and vice-versa. For example,

(i) $7856 = 7000 + 800 + 50 + 6$
$= (7 \times 1000) + (8 \times 100) + (5 \times 10) + 6$
$= (7 \times 10^3) + (8 \times 10^2) + (5 \times 10^1) + 6$

(ii) $(8 \times 10^5) + (2 \times 10^3) + (3 \times 10^2)$
$= (8 \times 10^5) + (0 \times 10^4) + (2 \times 10^3) + (3 \times 10^2) + (0 \times 10) + (0 \times 1)$
$= 802300.$

EXERCISE SET 2-16

Oral

1. (i) Give the base in 9^3.
   (ii) Give the exponent in 3^8.
2. Read the following :
   (i) 3^8. (ii) 7^3. (iii) 6^6. (iv) 5^2.

3. Give the following as powers of 10 :

(i) 1000000 (ii) 1000000000 (iii) 10 (iv) 100000.

4. Evaluate :

(i) (a) 3^2. (b) 4^2. (c) 5^2. (d) 6^2.
(ii) (a) 2^3. (b) 3^3. (c) 4^3.
(iii) (a) 1^2. (b) 1^3. (c) 0^2. (d) 0^3.

5 Read the following in *exponential notation* :

(i) 29. (ii) 30. (iii) 600. (iv) 8124. (v) 2004.

Written

6 Rewrite the following in symbols :

(i) Eight raised to the power five.
(ii) Hundred raised to the power two.
(iii) Ninth power of one thousand.
(iv) Tenth power of six.

7. Verify :

(i) $1^3 + 12^3 = 10^3 + 9^3$.
(ii) $3^2 + 4^2 = 5^2$.
(iii) $3^3 + 4^3 + 5^3 = 6^3$.

8. Express in decimal notation :

(i) $(3 \times 10^5) + (5 \times 10^2) + (8 \times 10) + 1$.
(ii) $(6 \times 10^4) + (4 \times 10^2) + 8$.
(iii) 4×10^5.
(iv) $(7 \times 10^6) + (2 \times 10^3)$.

9. Rewrite the following in exponential notation with 10 as base :

(i) $100 \times 10000 = 1000000$.
(ii) $10000 \times 1000 = 10000000$.

(iii) $10000000 \times 10 = 100000000$.
(iv) $10000 \times 10000 = 100000000$.

10. Rewrite the following in exponential notation :
(i) $100000 \div 100 = 1000$.
(ii) $1000 \div 10 = 100$.
(iii) $10000 \div 1000 = 10$.

2-17 Factors and Multiples

In this section we shall study some interesting properties of whole numbers. We shall come across challenging problems in our study. Many of these problems appear to be simple, but they still remain challenging.

Which numbers, when multiplied together, give the *product* 12 ? The answer can be given in different ways :

$$1 \times 12 = 12,$$
$$4 \times 3 = 12,$$
$$6 \times 2 = 12,$$
$$2 \times 2 \times 3 = 12.$$

We can make several observations on these statements :

4 is a *factor* of 12,
3 is a *factor* of 12,
4 and 3 are *divisors* of 12,
12 is a *multiple* of 4,
12 is a *multiple* of 3,

and so on.

Finding a set of whole numbers such that their continued product is the given whole number, is called *factorising* the given number.

If a number a is a *factor* or *divisor* of a number b, then a divides b with remainder zero.

If all the factors of a product are equal as in :

$$2 \times 2 = 4,$$
$$3 \times 3 \times 3 \times 3 = 81,$$
$$5 \times 5 \times 5 = 125, \text{ etc.},$$

then the product is a power of the factor, and if the number of equal factors is n, then we write

$$\text{number} = (\text{factor})^n,$$

and n is called the index of power. Thus,

4 is a second power of 2, and $4 = 2^2$,

81 is a fourth power of 3, and $81 = 3^4$,

125 is a third power of 5, and $125 = 5^3$,

and so on.

$a = b^n$ is the n^{th} power of b and is read as a is equal to b raised to the power n.

If the number of equal factors is two, the resulting number is called a *square* number or square. If it is three, it is called *cube*. For example,

4 is the square of 2, as $4 = 2 \times 2 = 2^2$,

9 is the square of 3, as $9 = 3 \times 3 = 3^2$,

16 is the square of 4, as $16 = 4 \times 4 = 4^2$;

and so on.

8 is the cube of 2, as $8 = 2 \times 2 \times 2 = 2^3$,

27 is the cube of 3, as $27 = 3 \times 3 \times 3 = 3^3$,

64 is the cube of 4, as $64 = 4 \times 4 \times 4 = 4^3$;

and so on.

Is 1 a square number ?

Since $1 \times 1 = 1$,

1 is a square number.

Similarly, 1 is a cube number also.

In fact $1^4 = 1$, $1^5 = 1$, and so on. 1 raised to any power is 1,

i. e., $1^n = 1$, n being any whole number.

Common Factor and Common Multiple

Can two or more whole numbers have the same factor ?

2 is a factor of 4 and 2 is also a factor of 6. We say 2 is a *common factor* of 4 and 6. Similarly, a whole number can be a common multiple of two or more whole numbers

12 is a multiple of 4 and 12 is also a multiple of 3. We, therefore, say that 12 is a common multiple of 4 and 3.

we observe that

$$12 \times 1 = 12$$
$$3 \times 1 = 3$$
$$4 \times 1 = 4.$$

In fact $a \times 1 = a$ for every whole number a.

Thus 1 is a factor of every whole number. Also every number is its own factor. These factors, 1 and the number itself, of the number are evidently *trivial factors* and the other factors are called *proper factors.* Thus 1 and 12, are the trivial factors of 12, while 2, 3, 4, 6 are all proper factors of 12.

EXERCISE SET 2-17

Oral

1. Give all the factors of each of the following numbers :

(i) 15. (ii) 42. (iii) 56. (iv) 63. (v) 70. (vi) 11.

2. 24 is expressed as a product of two factors. One of them is given below. Find the other factor :

(i) 4. (ii) 12. (iii) 3.

3. Are the following a multiple of 2 ?

(i) 8. (ii) 4. (iii) 16. (iv) 7. (v) 10. (vi) 1.

4. Give the *square* of the following :

(i) 6. (ii) 7. (iii) 9. (iv) 10. (v) 1.

5. Give the *cube* of the following :

(i) 3. (ii) 4. (iii) 6. (iv) 10. (v) 1.

6 Give all the *common factors* of each of the following pairs :

(i) 4 and 8. (ii) 6 and 9. (iii) 40 and 25.

7. Give all common multiples of each of the following pairs :

(i) 4 and 6. (ii) 3 and 6. (iii) 6 and 8. (iv) 8 and 10.

8. Express the following as powers :

(i) $3 \times 3 \times 3 \times 3 \times 3$.
(ii) $7 \times 7 \times 7 \times 7$.
(iii) $8 \times 8 \times 8 \times 8 \times 8 \times 8$.

9. Give the expansion of the following :

(i) 4^3. (ii) 9^4. (iii) 12^2.

Written

10. Factorise the following in as many ways as you can .

(i) 20. (ii) 48. (iii) 30.

11. Find a common multiple for the following :

    (i) 2, 5 and 6. (ii) 3, 4 and 5. (iii) 20, 40 and 50.

12. Find the common factors for the following :

    (i) 4, 12 and 20. (ii) 16, 40 and 56.

13. Find all the square numbers between 1 and 100.

14. Find all the cube numbers from 1 to 1000.

15. What digits occur and what digits do not occur in the units place of square numbers ?

2-18 Even and Odd Numbers

Consider whole numbers which are multiples of 2

0, 2, 4, 6, 8 and so on are all multiples of 2.

The whole numbers which are not multiples of 2, are

1, 3, 5, 7, 9 and so on.

The whole numbers which are multiples of 2 are called *even whole numbers* or simply *even numbers*. They form a subset of W. The whole numbers which are not multiples of 2 are called *odd numbers*. They also form another subset of W.

Consider the set of even numbers :

$$0, 2, 4, 6, 8, \ldots,$$

Is the sum of two even numbers an even number ?

Let us try a number of cases.

$$4 + 8 = 12,$$
$$2 + 6 = 8,$$
$$4 + 6 = 10,$$
$$2 + 2 = 4,$$
$$6 + 6 = 12.$$

In every case you choose, the sum of two even numbers is an even number. So we shall accept,

The set of even numbers is closed with respect to addition.

Now consider the set of odd numbers :

1, 3, 5, 7, ...,

Is the sum of two odd numbers an odd number ?

If we assume that it is so, we can easily give an example which shows that the assumption is not true. For example,

$$1 + 3 = 4$$

and 4 is not an odd number.

To show that a statement is not true for *all* cases, it is enough to give *one* example to the contrary, which is called a *counter example*

In fact

The sum of two odd numbers is never an *odd* number.

Is the sum of two odd numbers an even number ?

Let us try a number of cases.

$$1 + 3 = 4,$$

$$3 + 5 = 8,$$

$$1 + 7 = 8,$$

$$3 + 3 = 6,$$

$$5 + 5 = 10.$$

In every case we choose, we find that the sum of two odd numbers is an even number. Therefore,

The sum of any two odd numbers is even.

Is the product of two even numbers an even number ?

Let us try a number of cases.

$$\begin{gathered} 2 \times 4 = 8, \\ 6 \times 4 = 24, \\ 8 \times 2 = 16, \\ 4 \times 4 = 16, \\ 6 \times 6 = 36, \end{gathered}$$

and so on.

Thus in every case we choose, we find that the product of two even numbers is even. So we say,

The set of even numbers is closed with respect to multiplication.

Is the product of two odd numbers an odd number ?

We know

$$\begin{gathered} 1 \times 3 = 3, \\ 3 \times 7 = 21, \\ 5 \times 9 = 45, \\ 3 \times 3 = 9, \\ 5 \times 5 = 25, \end{gathered}$$

and so on.

In every case we choose, we find that the product of two odd numbers is an odd number and so we shall say,

The set of odd numbers is closed with respect to multiplication.

EXERCISE SET 2-18

Oral

1. Pick out the even numbers from the following :

   (i) 14. (ii) 24. (iii) 34. (iv) 241. (v) 342. (vi) 143.

2. Pick out the odd numbers from the following :

   (i) 315. (ii) 425. (iii) 335. (iv) 51 (v) 52. (vi) 5358.

Written

3. Give a *counter example* to show that each of the following statements is not true.

   (i) 3 is a factor of all whole numbers which end in three.

   (ii) The sum of an odd number and an even number is even.

   (iii) The difference of two odd numbers is odd.

   (iv) The product of an odd number and an even number is odd.

4. (i) Why is 0 an even number ?

   (ii) Given an even number, how will you find the next even number ?

   (iii) Given an odd number, how will you find the next odd number ?

5 Discover the pattern in each set. Write four more statements of the same pattern and state the property.

(i)

$$1 + 3 = 4,$$
$$1 + 3 + 5 = 9,$$
$$1 + 3 + 5 + 7 = 16,$$
$$1 + 3 + 5 + 7 + 9 = 25.$$

(ii)

$$1^2 - 0^2 = 1,$$
$$2^2 - 1^2 = 3,$$
$$3^2 - 2^2 = 5,$$
$$4^2 - 3^2 = 7.$$

6. Find the intersection of the set of even numbers and the set of odd numbers.

7. Are there the greatest even number and the greatest odd number ?

2-19 Prime and Composite Numbers

Number	Divisors
1	1
2	2, 1
3	3, 1
4	4, 2, 1
5	5, 1
6	6, 3, 2, 1
7	7, 1
8	8, 4, 2, 1
9	9, 3, 1
10	10, 5, 2, 1

continue the table upto 20.

What are the numbers which have more than 2 divisors ?

4, 6, 8, 9, 10 and so on have more than 2 divisors ?

What are the numbers which have only 2 divisors ?

2, 3, 5, 7 and so on have *only* 2 divisors.

The natural numbers which have only two divisors (which are trivial) are called the *prime numbers* or in short *primes.* The natural numbers which have more than two divisors are called the *composite numbers.* Since 1 has only one divisor, that is itself, it is neither prime, nor composite.

Consider a composite number, say 24. Factorise it. Suppose you factorise it as

$$24 = 8 \times 3.$$

But 8 can be factorised as 4×2,

so $$24 = 4 \times 2 \times 3.$$

Again 4 can be factorised as 2×2.

so $$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$

None of the factors can be factorised further.

What do you find ?

All the numbers in the product $2 \times 2 \times 2 \times 3$ are *prime numbers*. They are therefore called prime factors of 24.

Suppose you start with $24 = 6 \times 4$.

6 can be factorised as 3×2 and 4 as 2×2.

So $$24 = 3 \times 2 \times 2 \times 2.$$

We have again the same prime factors except for order.

Suppose you start with $24 = 12 \times 2$.

12 can be factorised as 4×3 and 4 as 2×2.

So $24 = 2 \times 2 \times 3 \times 2$, again the same prime factors except for order.

Thus we may start with any factors of a number, by successively factorising the *composite factors*, we will finally arrive at the same factorisation of the number. This is called the *prime factorisation* of the number.

Every composite number can be factorised into powers of primes in a unique way.

This is called the *fundamental theorem of Arithmetic.*

The prime numbers are therefore the building blocks of Arithmetic.

You can also find the prime factorisation as shown below. But you should be careful to start with prime 2, divide by it in as many stages as possible, then with 3, divide by it in as many stages as possible and so on, to avoid missing any prime factor

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

Thus,

$$24 = 2 \times 2 \times 2 \times 3$$

or $$24 = 2^3 \times 3.$$

Note : 1 is not given in the prime factorisation, as it is not a prime number. Moreover 1 is looked upon as a *trivial* factor of a number

It is no wonder that prime numbers have fascinated man ever since he began thinking of them. A number of questions have been asked about them. Only a few of them have been answered. Some of the questions are :

(1) How to find whether a given natural number is prime or not ?

(2) Is there a pattern in primes ?

(3) How to list as many primes as we want ?

Eratosthenes (276 – 194 B. C.), a Greek mathematician, suggested the simple and evident method of finding the prime numbers and it goes by the name of *Eratosthenes Sieve.*

Let us use the method to find prime numbers upto 100. Write 1 to 100 in an *array* as shown below :

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1 is not prime. Cross it. 2 is prime; round it. Starting from 2, cross out every second number.

~~1~~	②	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~
21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29	~~30~~
31	~~32~~	33	~~34~~	35	~~36~~	37	~~38~~	39	~~40~~
41	~~42~~	43	~~44~~	45	~~46~~	47	~~48~~	49	~~50~~
51	~~52~~	53	~~54~~	55	~~56~~	57	~~58~~	59	~~60~~
61	~~62~~	63	~~64~~	65	~~66~~	67	~~68~~	69	~~70~~
71	~~72~~	73	~~74~~	75	~~76~~	77	~~78~~	79	~~80~~
81	~~82~~	83	~~84~~	85	~~86~~	87	~~88~~	89	~~90~~
91	~~92~~	93	~~94~~	95	~~96~~	97	~~98~~	99	~~100~~

What multiples are crossed out ?

All the multiples of 2 or all even numbers are crossed out.

Again, 3 is prime; round it. Starting from 3, cross out every third number.

~~1~~	~~2~~	③	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	25	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~
~~51~~	~~52~~	53	~~54~~	55	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	65	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	77	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	85	~~86~~	~~87~~	~~88~~	89	~~90~~
91	~~92~~	~~93~~	~~94~~	95	~~96~~	97	~~98~~	~~99~~	~~100~~

What multiples are crossed out ?

All the multiples of 3 are crossed out.

We have so far considered 1, 2, and 3. Next number is 4.

But we find 4 already crossed out. Why ?

4 is composite. As multiple of 2, it is crossed out.

Is every fourth number from 4 crossed out ?

Yes, being a multiple of 2, these are already crossed out.

Take 5. It is prime. Round it. Cross out every fifth number from 5. All the multiples of five are crossed out. Take 6. It is already crossed out, being a multiple of 2 and 3. We see that every sixth number from 6 is crossed out already.

Take 7. It is the next prime. Round it. Cross out every seventh number from 7. All the multiples of 7 are crossed out.

Proceed thus and you end up with the following numbers, which are prime numbers from 1 to 100.

.	2	3		5	.	7	.	.	.
11	.	13	.	.	.	17	.	19	.
.	.	23	.	.	.	.	.	29	.
31	.	.	.	.	.	37	.	.	.
41	.	43	.	.	.	47	.	.	.
.	.	53	.	.	.	.	.	59	.
61	.	.	.	.	.	67	.	.	.
71	.	73	.	.	.	.	.	79	.
.	.	83	.	.	.	.	.	89	
.	.	.	.	.	.	97		.	.

The pairs (11, 13), (17, 19), (41, 43), (71, 73) etc have a difference of 2.

Prime numbers with a difference of 2 are called twin primes

When two natural numbers have 1 as the only common factor, they are said to be relatively prime to each other or co-prime.

For example : (6, 35), (8, 33) are pairs of numbers relatively prime to each other or are Co-primes.

EXERCISE SET 2-19

Oral

1. Give a counter example to show that each of the following statements is false :

   (i) The square of a prime number is always prime.

   (ii) A prime number is always odd.

(iii) A prime number does not end in 5.

(iv) A number which ends in 7 is prime.

(v) The sum of any two odd numbers is always prime.

2 Give two examples in support of the following statements :

(i) Two consecutive whole numbers are relatively prime to each other.

(ii) The sum of an odd number and an even number may be prime.

(iii) The sum of two composite numbers may be prime.

(iv) The difference of two composite numbers may be prime.

3. Are the set of prime numbers and the set of composite numbers disjoint ?

4. Give all the pairs of primes less than 100 and having a difference of 10

Written

5. Write four more statements on the following pattern ;

$$\begin{aligned} 2 + 1 &= 3, \\ (2 \times 3) + 1 &= 7, \\ (2 \times 3 \times 5) + 1 &= 31, \\ (2 \times 3 \times 5 \times 7) + 1 &= 211. \end{aligned}$$

6. Goldbach in 1742 in a letter to Euler, one of the greatest mathematicians, wrote that he had observed a pattern, which is reproduced as :

$$4 = 2 + 2$$

$$6 = 3 + 3$$

$$8 = 5 + 3$$
$$16 = 13 + 3$$
$$18 = 11 + 7$$

and so on.

Goldbach wanted to know if it would be correct to make the general statement :

'Every even number greater than 2, can be expressed as the sum of two primes'. Euler would not find a counter example to show it was false Neither could he prove it. Even to this day, it remains a *conjecture*.

Express the following as sum of two primes :

(i) 20 (ii) 22. (iii) 24. (iv) 256. (v) 300.

7. Goldbach made two other conjectures :

(i) Every even number not less than 6 is the sum of two odd primes

(ii) Every odd number not less than 9 is the sum of three odd primes.

Give two examples supporting each of the above statements.

8. Find *prime factorisation* of the following :

(i) 80 (ii) 36. (iii) 63. (iv) 150. (v) 384.
(vi) 257. (vii) 1729. (viii) 1111.

2-20 L.C.M. and H.C.F. of Natural Numbers

Take any two numbers, say 20 and 12. Write down the set of multiples of 20. (0 is excluded). We have

{ 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240,... }

Write down also the set of multiples of 12 We have

{ 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, ... }

Find their intersection, that is the set of the common multiples of 20 and 12. We have

$$\{ 60, 120, 180, \ldots \}$$

What is the least of the common multiples ?

It is 60. It is called the *least common multiple* (L.C.M.) of 20 and 12.

Now write down the set of factors of 20.

We have

$$\{ 1, 2, 4, 5, 10, 20 \}$$

Write down also the set of factors of 12.

We have

$$\{ 1, 2, 3, 4, 6, 12 \}$$

Find their intersection, that is, the set of the *common factors* of 20 and 12. We have

$$\{ 1, 2, 4 \}$$

What is the highest among the common factors ?

It is 4. It is called the *highest common factor* (H.C.F.) of 20 and 12.

To find the L.C M. and the H.C.F. of natural numbers, we use their prime factorisation. For example,

(i) $12 = 2\times2\times3 = 2^2\times3$,
$20 = 2\times2\times5 = 2^2\times5$.
L.C.M. of 12 and 20 $= 2\times2\times3\times5 = 60$.
H.C.F. of 12 and 20 $= 2\times2 = 4$.

(ii) $360 = 2^3\times3^2\times5$,
$225 = 3^2\times5^2$.

L.C.M. of 360 and 225 $= 2^3 \times 3^2 \times 5^2$,

H.C F. of 360 and 225 $= 3^2 \times 5$.

(iii) $45 = 3 \times 3 \times 5 = 3^2 \times 5$,

$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$,

$42 = 2 \times 3 \times 7$.

H.C.F. of 45, 120 and 42 $= 3$,

L.C.M. of 45, 120 and 42 $= 2^3 \times 3^2 \times 5 \times 7$.

We observe that

L.C.M. × H.C.F. = Product of the two numbers.

Euclidean Algorithm Process

H.C.F. can also be found out by a short-cut method called the Euclidean Algorithm. This method of finding the H.C.F. of two numbers was used by our ancient mathematician, Arya Bhatta, in the 5th century *A. D.* in solving a set of linear equations in integers. You will learn the proof of this method later. The method is explained below :

Let a and b be two whole numbers. If they are equal, either of them is their H.C.F.

Example : H.C.F. of 28 and 28 is 28.

If $a \neq b$, take the greater of them. Let it be a. Divide a by b. If the remainder is zero, then b is their H.C.F.

Example : H.C.F. of 28 and 14 is 14 as 14 divides 28.

If there is a remainder other than zero while dividing a by b, let the remainder be r_1. Now. $0 < r_1 < b$. Divide b by r_1. If the remainder is zero, then r_1 is the H.C.F. of a and b.

Example : Let us find the H.C.F. of 24 and 84.

Dividing 84 by 24, the remainder is 12. Now we take 12 and 24. 12 divides 24. So 12 is the H.C.F. of 84 and 24.

The process can be displayed as follows :

$$\begin{array}{rl} 24) & 84\ (3 \\ & \underline{72} \\ r_1 = & 12)\ 24\ (2 \\ & \qquad \underline{24} \\ & \qquad\ 0 \end{array}$$

If the remainder is not zero, while dividing b by r_1, let the remainder be r_2. Take r_1 and r_2. $0 < r_2 < r_1$. Divide r_1 by r_2 If the remainder is zero, then r_2 is the H.C.F. of a and b. Study the example worked out below.

Example · Let us find the H. C. F of 225 and 255 by Euclidean Algorithm process.

$$\begin{array}{rl} 225) & 255\ (1 \\ & \underline{225} \\ r_1 = & 30)\ 225\ (7 \\ & \qquad \underline{210} \\ r_2 = & \qquad 15)\ 30\ (2 \\ & \qquad\qquad \underline{30} \\ & \qquad\qquad\ 0 \end{array}$$

$\therefore$ 15 is the H.C.F. of 225 and 255.

If the remainder is not zero, while dividing r_1 by r_2, let it be r_3. Continue the process till the remainder is zero. The divisor at the stage which gives the remainder zero is the H.C.F. of a and b. Study the example worked out below.

Example : Find the H.C F. of 4296 and 9960 by the Euclidean Algorithm.

$$\begin{array}{l} 4296)\ 9960\ (2 \\ \qquad\ \underline{8592} \\ \qquad\ 1368)\ 4296\ (3 \\ \qquad\qquad\quad \underline{4104} \\ \qquad\qquad\quad 192)\ 1368\ (7 \\ \qquad\qquad\qquad\quad \underline{1344} \\ \qquad\qquad\qquad\quad 24)\ 192\ (8 \\ \qquad\qquad\qquad\qquad\quad \underline{192} \\ \qquad\qquad\qquad\qquad\qquad 0 \end{array}$$

$\therefore$ H.C.F. of 4296 and 9960 is 24.

EXERCISE SET 2-20

Oral

1. In finding the L.C.M. of two one-digit or small two digit numbers, there is an easy way. Think of, in succession, the multiples of the bigger number. Stop, on getting the first multiple divisible by the smaller number. It gives the L.C.M. of the two numbers; for example, to find L.C.M. of 6 and 8, the multiples of 8 are 8, 16, 24, etc. and 24 is divisible by 6. So 24 is the L.C.M. of 6 and 8.

Use the above technique and find the L.C.M. of the following

(i) 4, 6. (ii) 6, 9. (iii) 8, 12.
(iv) 10, 12. (v) 8, 10. (vi) 4, 8.
(vii) 16, 12. (viii) 8, 2. (ix) 20, 4. (x) 9, 12.

2. Find the H.C.F. of the following :

(i) 4, 6. (ii) 6, 9. (iii) 8, 12.
(iv) 10, 2. (v) 8, 10. (vi) 4, 8.
(vii) 16, 12. (viii) 8, 2. (ix) 20, 4. (x) 9, 12.

3. (i) Is the set of multiples of a natural number finite or infinite ?

   (ii) Is the set of divisors of a natural number finite or infinite ?

4. Find the H.C.F. of the following. Give your answer as a product of factors :

(i) $3 \times 3 \times 11, 3 \times 2 \times 11$.
(ii) $7 \times 3 \times 2, 5 \times 3 \times 5$.
(iii) $7 \times 5 \times 7, 3 \times 5 \times 3$.
(iv) $5 \times 5, 5 \times 5$.

5. Find the L.C.M. of the following. Give your answer as a product of factors :

(i) 2 × 2 × 5, 2 × 2 × 7.

(ii) 2 × 2 × 3, 2 × 2 × 2.

(iii) 2 × 3 × 5, 2 × 7 × 5.

(iv) 3 × 3 × 3, 3 × 3 × 3.

Written

6. Find the H.C.F. of the following. Give your answer as a product of factors :

(i) 3 × 2 × 5, 3 × 7 × 11, 3 × 11 × 13

(ii) $2^2 \times 3^2 \times 7$, $2^2 \times 3^2$, $2 \times 3^3 \times 5$.

7. Find the L.C.M. of the following. Give your answer as a product of factors.

(i) 2 × 3 × 5, 2 × 3 × 3 × 7, 2 × 2 × 7 × 13.

(ii) $2^2 \times 5$, $2^2 \times 7$, $2^3 \times 11 \times 7$.

8. Find by Euclidean Algorithm the H.C.F. of the following :

(i) 160, 248. (ii) 147, 1890. (iii) 225, 475.

9. Find the H.C.F. of the following :

(i) 15, 30, 36. (ii) 12, 24, 60. (iii) 20, 50, 100.

(iv) 5, 25, 75. (v) 40, 60, 80. (vi) 8, 12, 16, 36.

10. Find the L.C.M. of the following :

(i) 9, 15. (ii) 4, 5, 6. (iii) 12, 20, 22.

(iv) 4, 6, 8. (v) 250, 290. (vi) 306, 1173.

11 Complete the table and discover the pattern :

Number	Number	L.C M	H.C.F	Product of Numbers	Product of L C M and H C F
12	20	60	4	240	240
84	60	....	......	....	....
24	40	....	....	....	....
40	56	....	....	....	....
24	56	....	....	....	....

12. A school admitted 221 boys and 143 girls It was decided to divide them into the largest possible squads such that the number of boys in each boys' squad is the same as the number of girls in each girls' squad. Find the number of persons in each squad. Find also the number of boys' squads and the number of girls' squads.

13. I have two pieces of string, one 225 cm and the other 90 cm. If I want to cut them into equal bits of the longest possible size, find the size of each bit and the number of bits

14. Every fourth day in a hospital, a patient is given an injection and every third day tablets. On 2nd September he had both of them. On what other day in September he will have to take both of them

15. If the L.C.M. and the H.C F. of two whole numbers are the same, what can you say about the whole numbers ?

16. The L.C.M. and the H.C.F. of two numbers are respectively 160 and 8. If one of the numbers is 32, find the other.

3

THE SET OF FRACTIONAL NUMBERS

3-1 Introduction

We can easily see that if we subtract one whole number from another whole number, we do not always get a whole number; for example, subtracting 5 from 8 we get 3, which is a whole number, but if we wish to subtract 8 from 5, we do not get a whole number. Similar is the case of division of one whole number by another whole number.

Consider a few concrete situations :

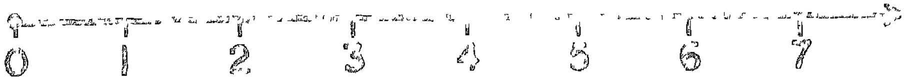
Suppose you are two brothers and your mother wants you to share equally what she gives you to eat. She gives 6 toffees. What do you do ? You divide by 2 and take $6 \div 2$, that is, 3 toffees. $6 \div 2$ can be written as $\frac{6}{2}$.

Again she gives you 5 biscuits. How will you divide them equally between you ? You have a problem. 5 is not divisible by 2. You therefore divide one of the biscuits into two halves, and each of you take two and a half biscuits. The 5 biscuits can now be divided into two equal parts. Each one of the parts is half of the 5 biscuits and we represent it by $\frac{5}{2}$.

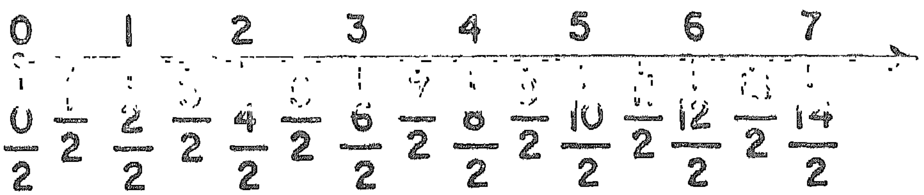
You could have divided the 5 biscuits equally between you in another way. Cut each of the biscuits into two equal parts, so that you have 10 bits. Now you can divide them equally between you. Each of you will get 5 bits out of ten bits.

So division in some concrete situations involves breaking up the whole into equal parts. Division, thus, takes a new meaning and gives rise to new numbers called *fractional numbers* or *positive rational numbers* as we shall call them later.

Take the number-ray representation of whole numbers. We take a set of equally spaced points starting with the initial point 0 of the number-ray and correspond them to the set of whole number in order.



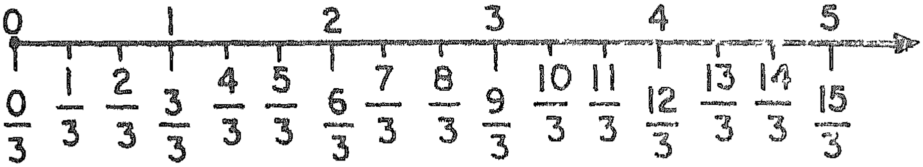
But there are other points on the line. Let us see, if some numbers can be associated with these or some of these points. Take the number ray marked with whole numbers. If we divide the distance between each two consecutive whole numbers in two equal parts, we get some more points on the number ray. These points together with points representing whole numbers are such that distance between any two consecutive points is the same. We mark these points, on the right of zero in succession as $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$, etc. We have new points as well as old points on the number ray marked as shown below.



Note that the unit is divided into two equal parts and the marking of points is now changed to fit the new sub-unit.

Again suppose we take the number-ray and mark the points anew in the following way :

We divide the distance between each two consecutive whole numbers in three (3) equal parts and mark the new points of subdivision, together with points associated with whole numbers in succession as $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}$, and so on. On number ray these points and the numbers corresponding to them are as shown below :



It may be noted that the unit is now divided into three equal parts and spacing of points is changed to fit the new sub-unit. We can repeat this process with 4, 5, 6, ... and get additional points in the ray marked with the corresponding numbers. Looking at the new numbers thus obtained, we find that these are expressed in terms of two whole numbers, one written above and the other below the dividing bar. The whole number above the dividing bar is called *numerator* and the whole number below the bar is called *denominator*. These new numbers are called *fractions* or *fractional numbers*. Number-ray hence forth will mean ray with points associated with fractional numbers.

In a fraction, denominator shows the kind of sub-unit considered and the numerator gives the number of such sub-units.

For example, consider $\frac{3}{5}$; here the sub-unit is *one-fifth* of the unit and there are *three-fifths* of the unit. Since a sub-unit can be obtained corresponding to any non-zero whole number, and any number of such sub-units can exist, therefore there is a fractional number with any whole number as numerator and any non-zero whole number as denominator. The set of all such fractions $\frac{m}{n}$, where m is any whole number and n is any *non-zero* whole number shall hereafter be denoted by Q_+, called *the set of fractional numbers or positive rational numbers.*

A fractional number is thus expressed in terms of an ordered pair of whole numbers a and b, (b≠o); it is written as $\frac{a}{b}$.

The whole numbers can also be expressed as fractional numbers, e.g.,

$$0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \ldots$$

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \ldots$$

$$2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \ldots$$

$$3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \ldots$$

Thus W can be considered as a subset of Q_+, i.e.,

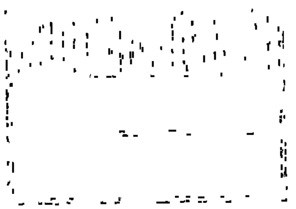
$$W \subset Q_+.$$

EXERCISE SET 3-1

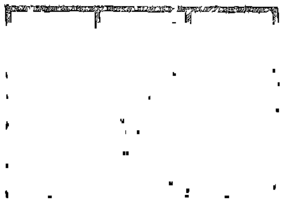
Oral

1. Associate fractional numbers to show what portions or regions are shaded below :

(i)

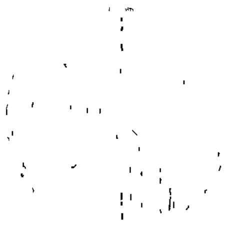


(ii)

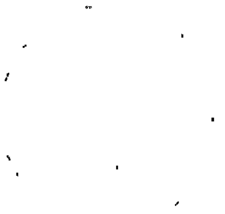


(iii)

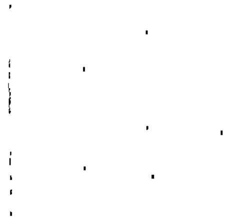
(iv)



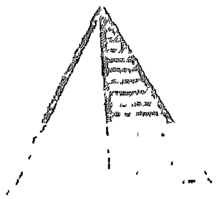
(v)



(vi)

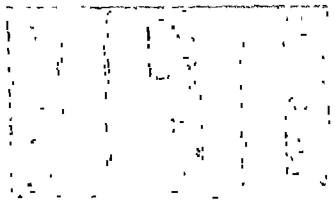


(vii)



(viii)

(ix)



2. State whether each of the following statements is true :

   (i) Every fractional number is expressed in terms of a whole number in the numerator and a natural number in the denominator.

   (ii) Every whole number can be expressed as a fractional number.

   (iii) Every fractional number is a whole number.

(iv) The set of whole numbers is a subset of the set of fractional numbers :

$$\left\{0, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \ldots\right\}$$

(v) The set of natural numbers is a subset of the set of fractional numbers :

$$\left\{0, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots\right\}.$$

3. Express the following fractional numbers as whole numbers :

(i) $\frac{28}{7}$. (ii) $\frac{36}{12}$. (iii) $\frac{0}{18}$. (iv) $\frac{50}{10}$. (v) $\frac{17}{17}$.

4. To what element in the set of fractional numbers :

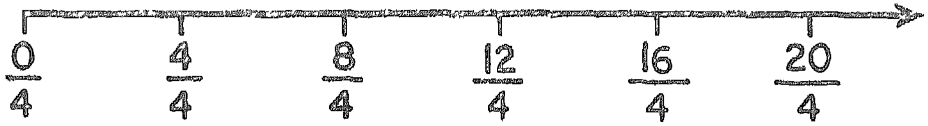
$$\left\{0, \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \ldots\right\},$$

is the whole number 3 equivalent ?

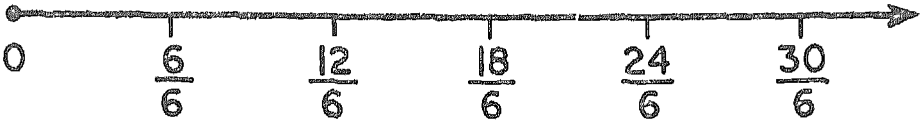
Written

5. Complete the sequences on the number rays given below by means of points for the numbers filling up the gaps.

(i)

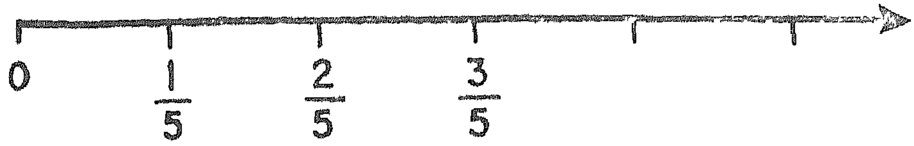


(ii)

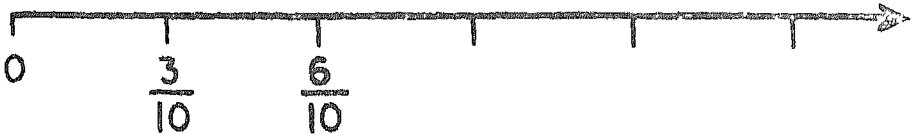


6. A few points on each number ray are associated with fractional numbers. Give fractional numbers for the other points on each number ray given below :

(i)

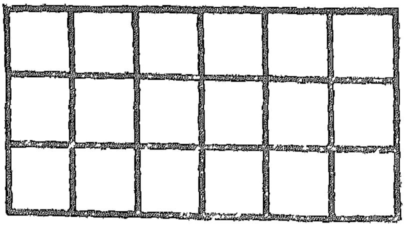


(ii)



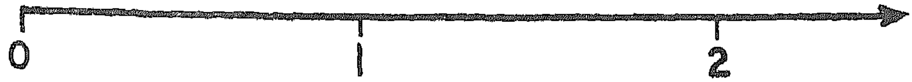
*7. Shade the portions in each of the regions given below to represent $\frac{1}{3}$. Give any two possible shadings in each case by drawing additional figures.

(i) (ii)



3-2 Equivalent fractions

Take a number ray with points marked 0, 1, 2.



Take 0, 1, 2, ... as

$$0, \frac{2}{2}, \frac{4}{2}, \ldots$$

$$0, \frac{3}{3}, \frac{6}{3}, \ldots$$

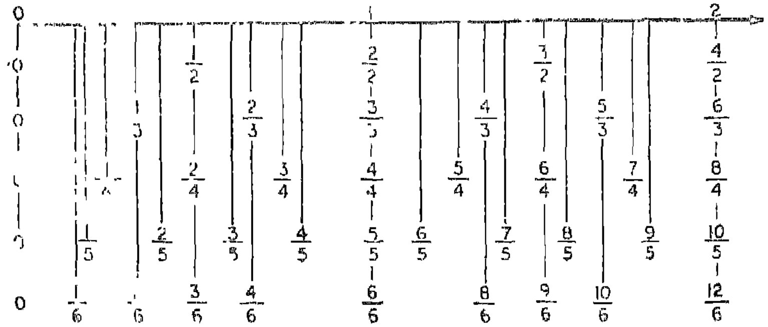
$$0, \frac{4}{4}, \frac{8}{4}, \ldots$$

$$0, \frac{5}{5}, \frac{10}{5}, \ldots$$

$$0, \frac{6}{6}, \frac{12}{6}, \ldots$$

to start with.

Associate points on the ray with fractional numbers that fill up the gaps in each representation of 0, 1, 2, ... You may make use of strips of paper equal in length to the unit chosen on the number-ray and fold them in order to get two equal parts, three equal parts, etc. This will help to fix halves, thirds, etc. We get the following figure:



This figure gives answers to a number of interesting questions. Let us take a few of them and a few others later.

If $\frac{1}{2}$ can be associated with a point, are there other fractions which can be associated with the same point ?

There are fractions $\frac{2}{4}$, $\frac{3}{6}$, etc., which can be associated with the same point. That means $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, etc. represent the same fractional number. So we can write

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \text{so on.}$$

Similarly,

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12},$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12},$$

$$\frac{2}{2} = \frac{3}{3} = \frac{4}{4},$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12},$$

$$\frac{4}{3} = \frac{8}{6} = \frac{12}{9},$$

and so on.

Fractions which represent the same fractional number are called equivalent fractions.

We would like to see how, given a fraction, we get other equivalent fractions.

Look for a pattern in the formation of equivalent fractions :

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12},$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12},$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}.$$

The pattern shows that the above equivalent fractions can be re-written as follows :

$$\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{3\times1}{3\times2}=\frac{4\times1}{4\times2}=\frac{5\times1}{5\times2}=\frac{6\times1}{6\times2}$$

$$\frac{1}{3}=\frac{2\times1}{2\times3}=\frac{3\times1}{3\times3}=\frac{4\times1}{4\times3}$$

$$\frac{2}{3}=\frac{2\times2}{2\times3}=\frac{3\times2}{3\times3}=\frac{4\times2}{4\times3}$$

So by continuing the process we can write as many equivalent fractions as we like, as shown below ·

$$\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{3\times1}{3\times2}=\frac{4\times1}{4\times2}=\frac{5\times1}{5\times2}=\frac{6\times1}{6\times2}=\frac{7\times1}{7\times2}=\frac{8\times1}{8\times2}=\text{etc.}$$

$$\frac{1}{3}=\frac{2\times1}{2\times3}=\frac{3\times1}{3\times3}=\frac{4\times1}{4\times3}=\frac{5\times1}{5\times3}=\frac{6\times1}{6\times3}=\frac{7\times1}{7\times3}=\frac{8\times1}{8\times3}=\text{etc.}$$

$$\frac{2}{3}=\frac{2\times2}{2\times3}=\frac{3\times2}{3\times3}=\frac{4\times2}{4\times3}=\frac{5\times2}{5\times3}=\frac{6\times2}{6\times3}=\frac{7\times2}{7\times3}=\frac{8\times2}{8\times3}=\text{etc.}$$

Thus we observe that :

$$\frac{a}{b}=\frac{m\times a}{m\times b}, \text{ where } m \in N.$$

In the set of equivalent fractions,

$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\},$$

$\frac{1}{2}$ is the *simplest form.* In the set of equivalent fractions,

$$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\},$$

$\frac{2}{3}$ is the simplest form

Now comes the question. When is a fraction in its simplest form ? Examining the numbers in numerator and denominator in each case, we find the following :

In $\frac{1}{2}$, 1 and 2 are relatively prime.

In $\frac{2}{4}$, 2 and 4 are *not* relatively prime.

In $\frac{3}{6}$, 3 and 6 are *not* relatively prime.

Similarly, in $\frac{2}{3}$, 2 and 3 are relatively prime.

In $\frac{4}{6}$, 4 and 6 are *not* relatively prime.

In $\frac{6}{9}$, 6 and 9 are *not* relatively prime.

So we observe :

A fraction is in its simplest form, when the whole numbers in its numerator and denominator are relatively prime.

By expressing the numerator and denominator of a fraction as two-factor product expressions with one factor common and removing the common factors repeatedly, we can find the equivalent fraction *in its simplest form,* for example :

$$\frac{12}{16} = \frac{2 \times 6}{2 \times 8},$$

$$\text{So } \frac{12}{16} = \frac{6}{8}.$$

But 6 and 8 are not relatively prime.

Again $\frac{6}{8} = \frac{2 \times 3}{2 \times 4}$. So $\frac{6}{8} = \frac{3}{4}$.

Now 3 and 4 are relatively prime; it means that the simplest equivalent fraction for $\frac{12}{16}$ is $\frac{3}{4}$. This can be shown by successive cancellations as follows :

$$\frac{12}{16} = \frac{\cancel{12}^{6}}{\cancel{16}_{8}} = \frac{\cancel{6}^{3}}{\cancel{8}_{4}} = \frac{3}{4}.$$

This process is also called transformation or reduction of a fraction to its simplest form.

We can get the reduction done in one step, by finding two-factor product expressions with *the highest common factor* for the numerator and the denominator, for example :

$$\frac{210}{168} = \frac{42 \times 5}{42 \times 4},$$

where 42 is the H.C.F. of 210 and 168.

Thus $\frac{210}{168}$ in its simplest equivalent form is $\frac{5}{4}$.

EXERCISE SET 3-2

Oral

1. Give five equivalent fractions for each of the following :

   (i) $\frac{1}{7}$. (ii) $\frac{3}{5}$. (iii) $\frac{2}{9}$. (iv) $\frac{7}{10}$.

2. Give a fraction equivalent to $\frac{1}{3}$ from the following set of fractions :

$$\left\{ \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \ldots \right\}.$$

3. Which of the following fractions have their numerators and denominators relatively prime ?

(i) $\frac{7}{9}$ · (ii) $\frac{8}{9}$ · (iii) $\frac{6}{9}$ · (iv) $\frac{6}{8}$ ·

(v) $\frac{6}{11}$ · (vi) $\frac{10}{12}$ · (vii) $\frac{49}{49}$ ·

Written

4. Fill in the following :

(i) $\frac{8}{10} = \frac{\square}{20}$ · (ii) $\frac{15}{\square} = \frac{5}{8}$·

(iii) $\frac{\square}{12} = \frac{12}{18}$· (iv) $\frac{10}{14} = \frac{15}{\square}$·

5. Reduce the following fractions to their simplest form :

(i) $\frac{60}{64}$ · (ii) $\frac{32}{36}$ · (iii) $\frac{36}{27}$ ·

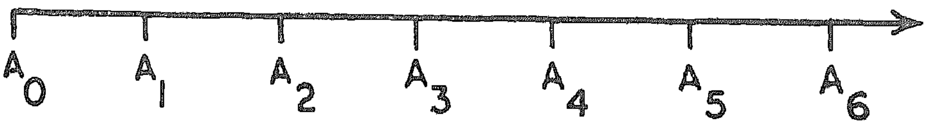
(iv) $\frac{124}{156}$ · (v) $\frac{357}{770}$ · (vi) $\frac{249}{492}$ ·

6. Complete the following to get true statements :

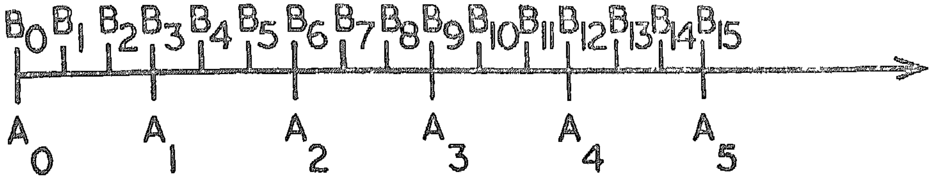
(i) By multiplying the numerator and the denominator of a fraction by the same non-zero whole number, ...fraction is obtained.

(ii) By dividing the numerator and the denominator of a fraction by the same non-zero whole number, ... fraction is obtained.

7. A_0, A_1, A_2, A_3, A_4, A_5 are points at equal intervals on the number-ray given below :



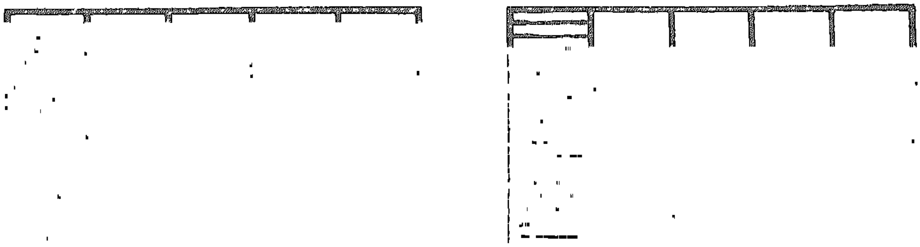
A_1 is associated with $\frac{1}{5}$. What fractional numbers will be associated with A_3 and A_5 ? If more points at different but equal intervals are taken on the number-ray as shown below :


B_0 B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8 B_9 B_{10} B_{11} B_{12} B_{13} B_{14} B_{15}
A_0 A_1 A_2 A_3 A_4 A_5

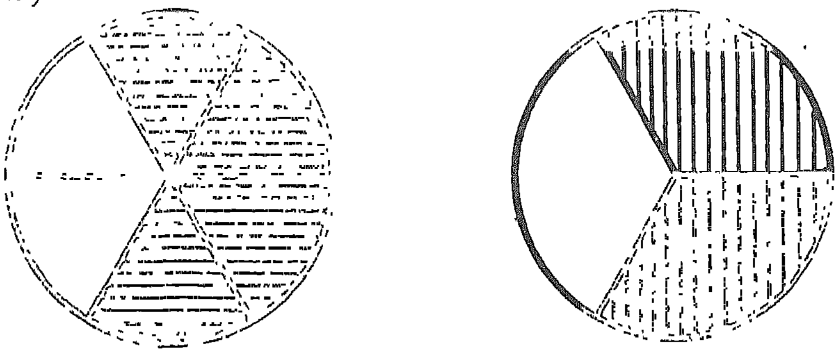
What fractional numbers will be associated with B_2, B_5, B_6, B_{11} and B_{12} ?

8. Write down the equivalent fractions represented by each pair of figures in the following :

(i)



(ii)



9. Give each of the following as fractions of the unit (given within brackets) in the simplest form.

   (i) 20 paise (rupee)

   (ii) 69 cm (metre)

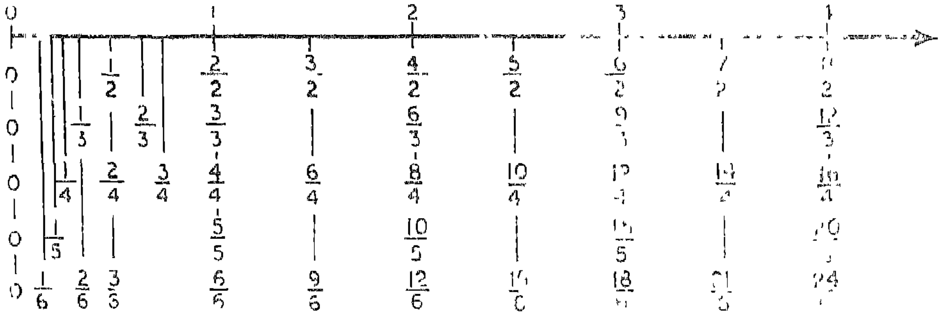
   (iii) 750 mg (gram)

*10. Depict by means of shading portions of rectangular regions of the same size

$$\frac{1}{4}=\frac{2}{8}=\frac{3}{12}.$$

3·3 Order

Take the number ray with points on it associated with fracional numbers as :



Taking any set of fractions with the same denominator, it is easy to tell which of any two of them is greater. For example in the set :

$$\left\{0, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \dots\right\},$$

$$\frac{2}{3}>\frac{1}{3},\ \frac{3}{3}>\frac{1}{3},\ \frac{4}{3}>\frac{1}{3},\ \frac{5}{3}>\frac{1}{3},$$

$$\frac{3}{3}>\frac{2}{3},\ \frac{4}{3}>\frac{2}{3},\ \frac{5}{3}>\frac{2}{3},$$

and so on

So,

If two fractions have the same denominator but different numerators, the fraction with the greater numerator is greater than the other fraction.

In symbols, if $\frac{m}{n}, \frac{p}{n} \in Q+$,

then $\frac{m}{n} > \frac{p}{n}$, if $m > p$.

Now arises the question : Given two fractions with the same numerators but different denominators, which of them is greater ?

Looking at the number-ray, we find

$$\frac{1}{2} > \frac{1}{3}, \frac{1}{2} > \frac{1}{4}, \frac{1}{2} > \frac{1}{5}, \ldots$$

$$\frac{1}{3} > \frac{1}{4}, \frac{1}{3} > \frac{1}{5}, \frac{1}{3} > \frac{1}{6}, \ldots$$

and so on.

Also $\frac{4}{3} > \frac{4}{4}, \frac{4}{3} > \frac{4}{5}, \ldots$

$$\frac{4}{4} > \frac{4}{5}, \frac{4}{4} > \frac{4}{6}, \ldots$$

$\frac{4}{5} > \frac{4}{6}$, and so on.

So,

If two fractions have the same numerators, but different denominators, the fraction with the lesser denominator is greater than the other fraction, i.e.,

if $\frac{a}{b}, \frac{a}{c} \in Q+$,

then $\frac{a}{b} > \frac{a}{c}$, *if* $b < c$.

The next question is to compare two fractions with different denominators and different numerators, e.g., $\frac{3}{6}$, $\frac{7}{8}$.

We have seen that it is easy to compare fractions when they have the same numerators or the same denominators. So we have to see if two fractions equivalent to $\frac{5}{6}$ and $\frac{7}{8}$ having the same numerator or denominator can be obtained.

In order to have the equivalent fractions of $\frac{5}{6}$ and $\frac{7}{8}$ with the same denominators, we have to find the product of 6 and 8, which is also one of the common multiples of the denominators 6 and 8. It is 48. Hence changing the denominators of both the fractions $\frac{5}{6}$ and $\frac{7}{8}$ to 48, we have the equivalent fractions as

$$\frac{5}{6} = \frac{5 \times 8}{6 \times 8} = \frac{40}{48} \text{ and } \frac{7}{8} = \frac{7 \times 6}{8 \times 6} = \frac{42}{48}$$

Instead of comparing $\frac{5}{6}$ and $\frac{7}{8}$, we can now compare their equivalent fractions $\frac{40}{48}$ and $\frac{42}{48}$, we easily see that $\frac{42}{48} > \frac{40}{48}$. That means $\frac{7}{8} > \frac{5}{6}$.

Now let us compare $\frac{7}{12}$ and $\frac{17}{20}$.

A common multiple of 12 and 20 is $12 \times 20 = 240$.

So $$\frac{7}{12} = \frac{7 \times 20}{12 \times 20} = \frac{140}{240}$$

and $$\frac{17}{20} = \frac{17 \times 12}{20 \times 12} = \frac{204}{240}$$

$$\therefore \quad \frac{17}{20} > \frac{7}{12}.$$

If we consider $\frac{1}{2}$ and $\frac{2}{4}$, these are equal as we have already seen.

Let us take another example :

compare $\frac{3}{8}$ and $\frac{9}{20}$.

We can compare these by making the numerators equal while finding equivalent fractions. For example, a common multiple of 3 and 9 is 9.

So $\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$ and $\frac{9}{20} = \frac{9 \times 1}{20 \times 1} = \frac{9}{20}$

This shows that $\frac{9}{24} < \frac{9}{20}$ and so $\frac{3}{8} < \frac{9}{20}$.

Here we note that when the numerators are the same, the fraction with the greater denominator is smaller or the fraction with the smaller denominator is greater. Hence

Given any two fractional numbers, there are only three possibilities, and only one of them is possible, namely :

(i) *the first one is greater than the second one;*

(ii) *the first one is equal to the second one;*

(iii) *the first one is less than the second one.*

We shall state the above three properties of order in symbols as follows :

Property 1

If $\frac{a}{b} \in Q_+$ and $\frac{c}{d} \in Q_+$,

then one and only one of the following statements is true

$$\frac{a}{b} > \frac{c}{d}, \quad \frac{a}{b} = \frac{c}{d}, \quad \frac{a}{b} < \frac{c}{d}.$$

Again take any three fractions with the same denominator, say,

$$\frac{5}{3}, \frac{2}{3} \text{ and } \frac{1}{3},$$

comparing them, we easily find that

$$\frac{5}{3} > \frac{2}{3}, \frac{2}{3} > \frac{1}{3} \text{ and } \frac{5}{3} > \frac{1}{3}$$

$$\text{or } \frac{1}{3} < \frac{2}{3}, \frac{2}{3} < \frac{5}{3} \text{ and } \frac{1}{3} < \frac{5}{3}.$$

Taking any three fractions with different denominators, say $\frac{3}{8}$, $\frac{5}{6}$ and $\frac{9}{20}$, we have to find first their equivalent fractions with the same denominator. A common multiple of 8, 6 and 20 is 120.

$$\text{So} \quad \frac{3}{8} = \frac{45}{120}, \frac{5}{6} = \frac{100}{120}, \frac{9}{20} = \frac{54}{120}.$$

$$\text{Thus } \frac{3}{8} < \frac{9}{20}, \frac{9}{20} < \frac{5}{6} \text{ and } \frac{3}{8} < \frac{5}{6}.$$

Property 2

If $\frac{a}{b} \in Q_+$, $\frac{c}{d} \in Q_+$ and $\frac{e}{f} \in Q_+$, *such that*

$$\frac{a}{b} < \frac{c}{d} \text{ and } \frac{c}{d} < \frac{e}{f}, \text{ then } \frac{a}{b} < \frac{e}{f}.$$

Also, if $\frac{a}{b} \in Q_+$, $\frac{c}{d} \in Q_+$ and $\frac{e}{f} \in Q_+$ *such that*

$$\frac{a}{b} > \frac{c}{d} \text{ and } \frac{c}{d} > \frac{e}{f}, \text{ then } \frac{a}{b} > \frac{e}{f}$$

There are a few interesting questions :

When does a fraction represent 1 ?

When does a fraction represent a fractional number less than 1 ?

When does a fraction represent a fractional number greater than 1 ?

Now $\frac{3}{3} = 1$, $\frac{7}{7} = 1$, $\frac{9}{9} = 1$ and so on.

When the numerator and the denominator of a fraction are the same non-zero whole number (or natural number) then the fraction represents 1.

Observing fractions representing fractional numbers *to the left of 1* on the number ray, say $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{7}$, $\frac{5}{6}$, etc., we find that their numerators are *less than* their denominators. *So*

If a fraction has its numerator less than its denominator, it represents a fractional number less than 1. Fractional numbers which are less than 1 are called proper fractional numbers.

In fractions like $\frac{3}{2}$, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{9}{6}$, etc., we find that their numerators are greater than their denominators and these fractions are on the right of 1 on the number-ray. *So*

If a fraction has its numerator greater than its denominator, it represents a fractional number greater than 1.

EXERCISE SET 3-3

Oral

1. Give the greater fractional number in each of the following pairs :

(i) $\frac{8}{9}$, $\frac{11}{9}$. (ii) $\frac{17}{13}$, $\frac{11}{13}$.

(iii) $\frac{7}{8}$, $\frac{7}{9}$. (iv) $\frac{9}{8}$, $\frac{9}{11}$.

2. Give the smaller fractional number in each of the following pairs :

(i) $\frac{3}{7}, \frac{4}{7}$. (ii) $\frac{5}{6}, \frac{2}{6}$.

(iii) $\frac{5}{5}, \frac{5}{3}$. (iv) $\frac{11}{6}, \frac{11}{13}$.

3. Say if each of the following represents a proper fractional number :

(i) $\frac{5}{6}$. (ii) $\frac{6}{5}$. (iii) $\frac{4}{9}$. (iv) $\frac{4}{3}$.

(v) $\frac{7}{7}$. (vi) $\frac{3}{7}$. (vii) $\frac{9}{7}$. (viii) $\frac{11}{19}$.

4. Give two proper fractional numbers.

5. Answer the following in terms of fractional numbers (without expressing them in their simplest forms).

(i) 5 bananas cost 41 paise. What is the price of one banana ?

(ii) 2 note-books cost 60 paise. What is the price of one note-book ?

Written

6. Which fractional number is greater in each of the following pairs ?

(i) $\frac{8}{12}, \frac{10}{18}$. (ii) $\frac{16}{20}, \frac{10}{15}$. (iii) $\frac{7}{8}, \frac{9}{10}$.

(iv) $\frac{9}{5}, \frac{4}{7}$. (v) $\frac{11}{3}, \frac{12}{5}$. (vi) $\frac{4}{5}, \frac{5}{11}$.

7. Which is cheaper ?

(i) 5 bananas for 33 paise or 4 bananas for 25 paise.

(ii) 7 oranges for 75 paise or 9 oranges for 100 paise.

(iii) 11 books for Rs. 35 or 12 books for Rs. 36.

8. From the pattern shown below, state the general properties in each of the following :

Give your answer in symbols (a, b, c, d).

(i) $\frac{6}{8} = \frac{9}{12}$ and $6 \times 12 = 8 \times 9$,

$\frac{10}{15} = \frac{6}{9}$ and $10 \times 9 = 6 \times 15$,

$\frac{5}{6} = \frac{20}{24}$ and $5 \times 24 = 6 \times 20$.

(ii) $\frac{5}{7} > \frac{3}{8}$ and $5 \times 8 > 7 \times 3$,

$\frac{6}{11} > \frac{2}{5}$ and $6 \times 5 > 11 \times 2$,

$\frac{10}{3} > \frac{13}{5}$ and $10 \times 5 > 13 \times 3$.

(iii) $\frac{2}{5} < \frac{3}{7}$ and $2 \times 7 < 5 \times 3$,

$\frac{6}{5} < \frac{11}{6}$ and $6 \times 6 < 11 \times 5$,

$\frac{7}{12} < \frac{8}{11}$ and $7 \times 11 < 12 \times 8$.

9. Compare $\frac{8}{5}$ and $\frac{7}{8}$ by means of their order relationship to 1.

10. For each pair of fractions given below, find a third fraction with its numerator equal to the sum of the numerators of the fractions and its denominator equal to the sum of the denominators of the fractions.

Arrange the triplets in ascending order of magnitude.

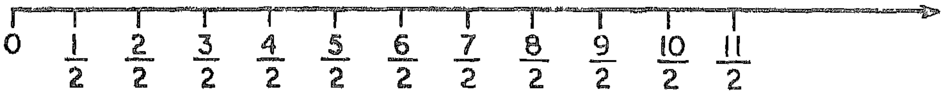
(i) $\frac{1}{2}, \frac{1}{3}$. (ii) $\frac{2}{3}, \frac{5}{6}$.

(iii) $\frac{5}{3}, \frac{3}{2}$. (iv) $\frac{7}{4}, \frac{2}{5}$.

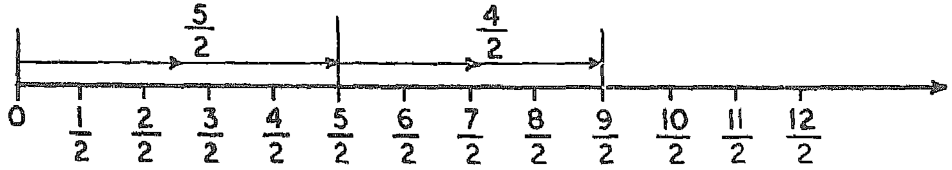
Can you state the corresponding general property ?

3-4 Addition in Q_+

Consider the number ray with points associated with fractional numbers as shown below :



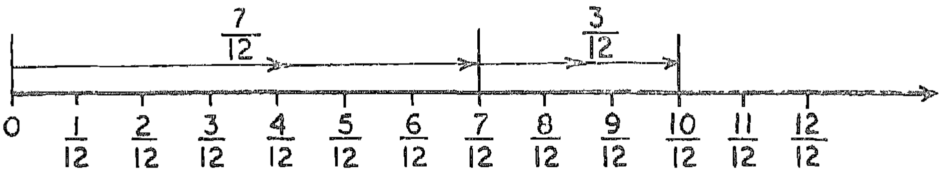
As in the set of whole numbers, we can add any two fractional numbers shown on the number ray. The addition of $\frac{5}{2}$ and $\frac{4}{2}$ is shown in the following figure :



This gives

$$\frac{5}{2} + \frac{4}{2} = \frac{9}{2}.$$

Similarly, adding $\frac{7}{12}$ and $\frac{3}{12}$,



we get

$$\frac{7}{12} + \frac{3}{12} = \frac{10}{12}.$$

In the above examples, the fractional numbers are such that their denominators are equal. We find that given two fractional numbers $\frac{a}{b}$ and $\frac{c}{b}$, their sum is $\frac{a+c}{b}$. We therefore write

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}.$$

We now take the next question; how to add two fractional numbers represented by fractions with different denominators ? You have already seen that fractions can be transformed into equivalent fractions with the same denominator ?

For example, to find

$$\frac{7}{12} + \frac{11}{9},$$

we first express $\frac{7}{12}$ and $\frac{11}{9}$ into equivalent fractions with the same denominator. The product of 12 and 9 is 108.

Thus

$$\frac{7}{12}=\frac{7\times 9}{12\times 9}=\frac{63}{108} \text{ and } \frac{11}{9}=\frac{11\times 12}{9\times 12}=\frac{132}{108},$$

$$\text{so } \frac{7}{12}+\frac{11}{9}=\frac{63}{108}+\frac{132}{108}=\frac{63+132}{108}=\frac{195}{108}.$$

Expressing $\frac{195}{108}$ in its simplest form, we have

$$\frac{195}{108}=\frac{65\times 3}{36\times 3}=\frac{65}{36}.$$

Thus $$\frac{7}{12}+\frac{11}{9}=\frac{65}{36}.$$

What would we get if we take L C.M. of 12 and 9 ?

L.C.M. of 12 and 9 is 36.

$$\text{So } \frac{7}{12}=\frac{7\times 3}{12\times 3}=\frac{21}{36} \text{ and } \frac{11}{9}=\frac{11\times 4}{9\times 4}=\frac{44}{36}.$$

$$\text{Thus } \frac{7}{12}+\frac{11}{9}=\frac{21}{36}+\frac{44}{36}=\frac{21+44}{36}=\frac{65}{36}.$$

Note that this sum is in its simplest form.

Symbolically to add two fractional numbers $\frac{a}{b}$ and $\frac{c}{d}$ with different denominators, we first find their equivalent fractions, with the same denominator.

A common multiples of b and d is bd.

So $$\frac{a}{b}=\frac{ad}{bd} \text{ and } \frac{c}{d}=\frac{bc}{bd}.$$

Thus $$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}.$$

How to add 1 *and* $\frac{3}{4}$ *?*

We can express 1 as a fractional number with denominator 4

$$\frac{1}{1} = \frac{1\times4}{1\times4} = \frac{4}{4},$$

so $$1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}.$$

It is usual to write $1 + \frac{3}{4}$ as $1\frac{3}{4}$ (read as 1 and $\frac{3}{4}$ or one whole number three by four).

Thus $\frac{7}{4}$ and $1\frac{3}{4}$ represent the same fractional number.

How to add 3 *and* $\frac{7}{5}$ *?*

We can express 3 as a fractional number with the denominator 5.

$$\frac{3}{1} = \frac{3 \times 5}{1 \times 5} = \frac{15}{5}.$$

So $$3+\frac{7}{5}=\frac{15}{5}+\frac{7}{5}=\frac{22}{5}=\frac{4\times5+2}{5}=\frac{4\times5}{5}+\frac{2}{5}=4+\frac{2}{5}.$$

It is usual to write $4 + \frac{2}{5}$ as $4\frac{2}{5}$ (read as 4 and $\frac{2}{5}$)

Then $\frac{22}{5}$ and $4\frac{2}{5}$ represent the same fractional number.

Proper and improper fractional numbers

As already stated a fractional number whose numerator is less than the denominator is called a *proper fraction*, for example $\frac{3}{5}$; while a fractional number whose numerator is greater than th denominator is called an improper fraction, for example $\frac{7}{5}$. Thus a proper fraction is less than 1 and an improper fraction is greater than 1. An improper fractional number can be written as the sum of a whole number and a proper fractional number.

EXERCISE SET 3-4

Oral

1. What is the sum of each of the following ?

(i) $\frac{3}{4}+\frac{5}{4}$. (ii) $2+\frac{2}{3}$. (iii) $3+\frac{7}{4}$.

(iv) $\frac{5}{6}+\frac{7}{8}$. (v) $\frac{7}{3}+\frac{2}{3}$. (vi) $4+\frac{5}{6}$.

(vii) $\frac{8}{12}+\frac{9}{4}$. (viii) $\frac{3}{4}+\frac{7}{6}$. (ix) $\frac{1}{7}+\frac{3}{11}$.

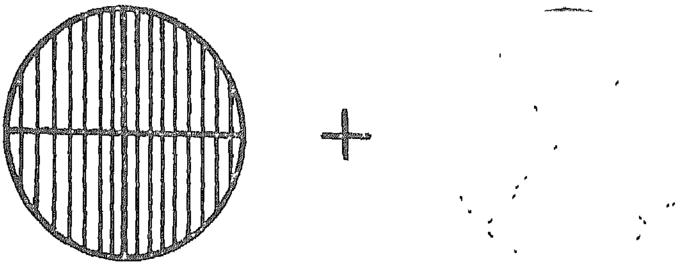
2. Fill in the following :

(i) $4=\frac{\square}{8}$. (ii) $7=\frac{\square}{3}$.

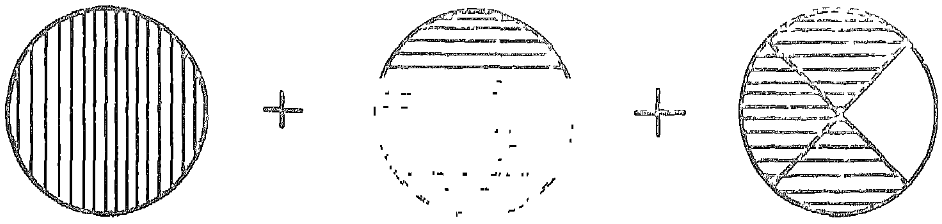
(iii) $\frac{\square}{5}=5$ (iv) $3=\frac{\square}{4}=\frac{\triangle}{6}$.

3. Give the fractional numbers represented by the following diagrams and write them in two different ways.

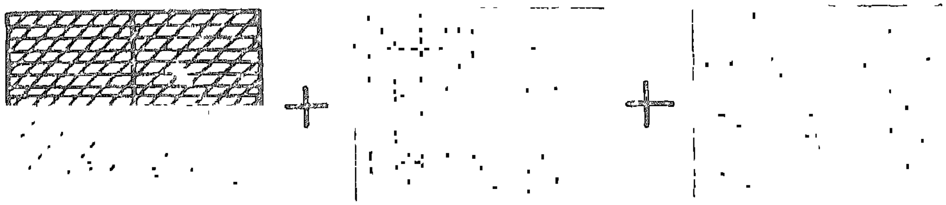
(i)



(ii)



(iii)



4. Fill in the following :

(i) $\frac{2}{7} + \square = \frac{5}{7}$. (ii) $\frac{2}{7} + \square = \frac{9}{7}$.

(iii) $\frac{2}{7} + \square = \frac{9}{14}$. (iv) $\frac{2}{7} + \square = 2$.

(v) $\frac{2}{7} + \square = \frac{2}{7}$. (vi) $\frac{2}{7} + \square = 1$.

Written

5. Express the following as the sum of a whole number and a proper fraction :

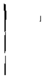
(i) $\frac{20}{7}$. (ii) $\frac{28}{5}$. (iii) $\frac{30}{11}$.

6. Express the following into improper fractions :

(i) $4\frac{2}{7}$. (ii) $3\frac{3}{5}$. (iii) $5\frac{7}{8}$. (iv) $8\frac{9}{11}$.

*7. Each of the following diagrams is part of a whole. What part it forms of the whole is indicated against each. Copy out and complete the diagram to represent the whole.

(i) $\frac{1}{4}$

(ii)  $\frac{3}{4}$

8. Simplify :

(i) $\frac{7}{20}+\frac{8}{15}$. (ii) $\frac{5}{12}+\frac{13}{8}$. (iii) $\frac{8}{12}+\frac{9}{16}$.

(iv) $\frac{12}{15}+\frac{20}{25}$. (v) $\frac{29}{18}+\frac{31}{24}$. (vi) $\frac{43}{24}+\frac{19}{10}$.

9. Replace □ by any fractional number in the following and state if the statement remains true in each case.

(i) $\frac{2}{3}>\frac{1}{2}$ implies $\frac{2}{3}+\square>\frac{1}{2}+\square$.

(ii) $\frac{3}{5}<\frac{5}{8}$ implies $\frac{3}{5}+\square<\frac{5}{8}+\square$.

(iii) $\frac{4}{6}=\frac{6}{9}$ implies $\frac{4}{6}+\square=\frac{6}{9}+\square$.

10. If $\frac{p}{q}$ is denoted as the number pair (p, q),

(a) Denote (i) $\frac{7}{4}$, (ii) $\frac{3}{2}$, (iii) $\frac{8}{11}$ as number pairs.

(b) What whole numbers are represented by

(i) (6, 3), (ii) (12, 2), (iii) (7, 7), (iv) (1, 1) ?

(c) Express 8 as a number pair.

(d) Find each of the following :

(i) (7, 3) + (5, 3). (ii) (2, 5) + (1, 5).

(iii) (3, 10) + (13, 15). (iv) (a, b) + (c, d).

(v) (a,b) + (a, c)

(e) Is (a, b) = (b, a) always ? Explain.

3-5 Properties of Addition in Q_+

Commutative Property

We would like to see if addition of two fractional numbers is commutative. Consider the following examples :

(i) $\frac{2}{3} + \frac{5}{8} = \frac{31}{24}$ and $\frac{5}{8} + \frac{2}{3} = \frac{31}{24}$;

so $\frac{2}{3} + \frac{5}{8} = \frac{5}{8} + \frac{2}{3}$.

(ii) $\frac{7}{12} + \frac{11}{9} = \frac{195}{108}$ and $\frac{11}{9} + \frac{7}{12} = \frac{195}{108}$,

so $\frac{7}{12} + \frac{11}{9} = \frac{11}{9} + \frac{7}{12}$.

We, therefore, observe that

Given any two fractional numbers $\frac{a}{b}$ *and* $\frac{c}{d}$,

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}.$$

Addition in Q_+ *satisfies commutative property.*

Associative Property

(i) Let us find $\frac{2}{3} + \frac{5}{6} + \frac{7}{8}$ by different grouping.

$$\left(\frac{2}{3} + \frac{5}{6}\right) + \frac{7}{8} = \frac{9}{6} + \frac{7}{8} = \frac{57}{24},$$

$$\frac{2}{3} + \left(\frac{5}{6} + \frac{7}{8}\right) = \frac{2}{3} + \frac{41}{24} = \frac{57}{24};$$

so $\left(\frac{2}{3}+\frac{5}{6}\right)+\frac{7}{8}=\frac{2}{3}+\left(\frac{5}{6}+\frac{7}{8}\right).$

(ii) Let us find $\frac{7}{2}+\frac{3}{4}+\frac{5}{8}$ by different grouping.

$$\left(\frac{7}{2}+\frac{3}{4}\right)+\frac{5}{8}=\frac{17}{4}+\frac{5}{8}=\frac{39}{8},$$

$$\frac{7}{2}+\left(\frac{3}{4}+\frac{5}{8}\right)=\frac{7}{2}+\frac{11}{8}=\frac{39}{8};$$

so $\left(\frac{7}{2}+\frac{3}{4}\right)+\frac{5}{8}=\frac{7}{2}+\left(\frac{3}{4}+\frac{5}{8}\right).$

We have therefore,

Given any three fractional numbers $\frac{a}{b}$, $\frac{c}{d}$ *and* $\frac{e}{f}$,

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right).$$

Addition in Q_+ satisfies associative property

As in the case of whole numbers, we can rearrange the terms in any convenient manner in a sum of fractional numbers.

Example : Find the sum : $\frac{7}{2}+\frac{3}{4}+\frac{3}{2}+\frac{1}{4}$.

$$\begin{aligned}\frac{7}{2}+\frac{3}{4}+\frac{3}{2}+\frac{1}{4}&=\frac{7}{2}+\frac{3}{2}+\frac{3}{4}+\frac{1}{4}\\&=\left(\frac{7}{2}+\frac{3}{2}\right)+\frac{3}{4}+\frac{1}{4}\\&=\frac{10}{2}+\frac{3}{4}+\frac{1}{4}\\&=\frac{10}{2}+\left(\frac{3}{4}+\frac{1}{4}\right)\\&=\frac{10}{2}+\frac{4}{4}\\&=5+1\\&=6.\end{aligned}$$

We can do this also in fewer steps as follows :

$$\frac{7}{2} + \frac{3}{4} + \frac{3}{2} + \frac{1}{4} = \left(\frac{7}{2} + \frac{3}{2}\right) + \left(\frac{3}{4} + \frac{1}{4}\right)$$
$$= 5 + 1$$
$$= 6$$

Note that several of the basic properties of W hold for Q_+ also.

Additive Identity

0 is a whole number. It can also be expressed as a fractional number

$$0 = \frac{0}{4}, \quad 0 = \frac{0}{5}, \quad 0 = \frac{0}{8}, \quad 0 = \frac{0}{1} \text{ and so on.}$$

Let us find (1) $0 + \frac{3}{4}$

$$0 + \frac{3}{4} = \frac{0}{4} + \frac{3}{4} = \frac{0+3}{4} = \frac{3}{4}$$

(2) $\frac{7}{5} + 0$

$$\frac{7}{5} + 0 = \frac{7}{5} + \frac{0}{5} = \frac{7+0}{5} = \frac{7}{5}.$$

So we shall note that for any fractional number $\frac{a}{b}$,

$$\frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}.$$

Therefore we have

0 is the additive identity in the set of fractional numbers.

We give below some solved examples.

Example 1 : Find $1\frac{1}{3} + 2\frac{2}{3}$.

Solution : $1\frac{1}{3} + 2\frac{2}{3} = \left(1 + \frac{1}{3}\right) + \left(2 + \frac{2}{3}\right)$

$$= (1 + 2) + \left(\frac{1}{3} + \frac{2}{3}\right)$$
$$= 3 + 1 = 4.$$

Example 2 : Find $3\frac{1}{2} + 4\frac{2}{3} + 7\frac{5}{6}$.

Solution : The sum is

$$= (3 + 4 + 7) + \left(\frac{1}{2} + \frac{2}{3} + \frac{5}{6}\right)$$

$$= 14 + \left(\frac{3}{6} + \frac{4}{6} + \frac{5}{6}\right)$$

$$= 14 + \frac{12}{6} = 14 + 2$$

$$= 16.$$

Example 3 : Find $3\frac{1}{8} + 4\frac{5}{12} + \frac{5}{6}$.

Solution :

$$3\frac{1}{8} + 4\frac{5}{12} + \frac{5}{6}$$

$$= \frac{25}{8} + \frac{53}{12} + \frac{5}{6}$$

$$= \frac{75}{24} + \frac{106}{24} + \frac{20}{24}$$

$$= \frac{201}{24} = \frac{67 \times 3}{8 \times 3} = \frac{67}{8} = 8\frac{3}{8}.$$

EXERCISE SET 3-5

Oral

1. Simplify :

(i) $\frac{8}{5} + 2 + \frac{2}{5}$.

(ii) $\frac{7}{3} + 1 + \frac{5}{3}$.

(iii) $\frac{1}{4} + \frac{1}{2} + \frac{1}{4}$.

(iv) $\frac{3}{8} + \frac{1}{2} + \frac{1}{8}$.

(v) $1\frac{1}{2} + 3\frac{1}{2}$.

(vi) $7\frac{3}{4} + 2\frac{1}{4}$.

(vii) $8\frac{1}{2} + \frac{1}{4} + 1\frac{1}{4}$.

(viii) $3\frac{1}{6} + 1\frac{2}{3} + 2\frac{1}{6}$.

2 Simplify :

(i) $1\frac{1}{3} + 3\frac{1}{3}$. (ii) $4\frac{1}{8} + 3\frac{3}{8}$.

(iii) $2\frac{21}{32} + 14\frac{23}{32}$. (iv) $30 + 7\frac{4}{5}$.

(v) $9\frac{2}{3} + 5$. (vi) $0 + 2\frac{3}{4} + 5$.

Written

3. Simplify :

(i) $\frac{19}{20} + 1\frac{2}{5} + 4\frac{7}{10}$. (ii) $\frac{3}{5} + 1\frac{4}{15} + 1\frac{2}{3}$.

(iii) $2\frac{3}{4} + \frac{7}{8} + 3$. (iv) $\frac{1}{5} + \frac{5}{6} + \frac{4}{5} + \frac{1}{6}$

4. Simplify ·

(i) $3\frac{7}{11} + 0 + 5\frac{3}{22}$. (ii) $\frac{1}{2} + \frac{1}{40} + \frac{7}{8} + \frac{3}{16}$.

(iii) $\frac{1}{3} + \frac{5}{96} + \frac{1}{12} + \frac{1}{24}$.

5. Find $3\frac{5}{11} + 2\frac{9}{13} + 5\frac{3}{5}$.

6. Simplify $4\frac{6}{7} + 5\frac{2}{3} + 3 + 8\frac{9}{11}$.

3-6 Subtraction in Q_+

You have learnt in W that subtraction is an operation inverse to that of addition and to every addition there are two subtractions. We shall see how these are extended in Q_+.

Consider the additions :

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7} \quad \Big| \quad \frac{1}{2} + \frac{3}{5} = \frac{11}{10}.$$

What are the subtractions that can be got from each of them ? They are :

$$\begin{array}{ll|ll} & \frac{5}{7}-\frac{2}{7}=\frac{3}{7}, & & \frac{11}{10}-\frac{3}{5}=\frac{1}{2}, \\ \text{and} & \frac{5}{7}-\frac{3}{7}=\frac{2}{7} & \text{and} & \frac{11}{10}-\frac{1}{2}=\frac{3}{5}. \end{array}$$

So you have two subtractions for every addition in the case of fractional numbers also.

You have also learnt that to add two fractional numbers with different denominators we transform the fractions representing them in equivalent fractions with the same denominator and then add We do the same thing in subtracting a fraction from another with different denominators. For example,

$$\frac{11}{10}-\frac{3}{5}=\frac{11}{10}-\frac{6}{10}=\frac{5}{10}=\frac{1}{2},$$

$$\frac{11}{10}-\frac{1}{2}=\frac{11}{10}-\frac{5}{10}=\frac{6}{10}=\frac{3}{5},$$

$$\frac{3}{4}-\frac{5}{7}=\frac{3\times7}{4\times7}-\frac{5\times4}{7\times4}=\frac{21}{28}-\frac{20}{28}=\frac{21-20}{28}=\frac{1}{28}.$$

Symbolically,

$$\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}.$$

You have further learnt that in W, subtraction is always not possible. We can subtract 3 from 5, i. e., 5—3 is possible, but we cannot subtract 11 out of 7 or 13 out of 10 and so on, i.e., we cannot find $a-b$ in W if $b>a$; therefore it would not be possible for us to find $\frac{a}{b}-\frac{c}{d}$ which is equivalent to $\frac{ad-bc}{bd}$, if $bc > ad$. Thus we observe that in Q_+, $\frac{a}{b}-\frac{c}{d}$ is possible only if $ad \geqslant bc$.

Since $\frac{a}{b} = \frac{ad}{bd}$ and $\frac{c}{d} = \frac{bc}{bd}$, this amounts to the fact that we can subtract $\frac{c}{d}$ from $\frac{a}{b}$ if and only if $\frac{a}{b} \geqslant \frac{c}{d}$.

Examples

(i) Find $1 - \frac{5}{12}$.

Solution : $1 - \frac{5}{12} = \frac{12}{12} - \frac{5}{12} = \frac{7}{12}$.

(ii) Find $3 - \frac{7}{9}$.

Solution .
$$\begin{aligned} 3 - \frac{7}{9} &= \frac{3}{1} - \frac{7}{9} \\ &= \frac{27}{9} - \frac{7}{9} \\ &= \frac{20}{9} \\ &= 2\frac{2}{9}. \end{aligned}$$

(iii) Find $6\frac{2}{3} - 4\frac{1}{2}$.

Solution :
$$\begin{aligned} 6\frac{2}{3} - 4\frac{1}{2} &= 6 + \frac{2}{3} - \left(4 + \frac{1}{2}\right) \\ &= 6 - 4 + \frac{2}{3} - \frac{1}{2} \\ &= 2 + \frac{4}{6} - \frac{3}{6} \\ &= 2 + \frac{1}{6} \\ &= 2\frac{1}{6}. \end{aligned}$$

(iv) Find $5\frac{3}{8} - 3\frac{5}{6}$.

Solution : $5\frac{3}{8} - 3\frac{5}{6} = \frac{43}{8} - \frac{23}{6}$

$$= \frac{129 - 92}{24}$$

$$= \frac{37}{24}$$

$$= 1\frac{13}{24}.$$

(v) Find $4 - \frac{3}{7}$.

Solution : $4 - \frac{3}{7} = \frac{4 \times 7}{7} - \frac{3}{7}$

$$= \frac{28 - 3}{7} = \frac{25}{7}$$

$$= 3\frac{4}{7}.$$

EXERCISE SET 3-6

Oral

1. Find :

(i) $\frac{3}{9} - \frac{1}{9}$. (ii) $\frac{11}{7} - \frac{2}{7}$. (iii) $\frac{5}{8} - \frac{5}{8}$

(iv) $1 - \frac{1}{6}$. (v) $1 - \frac{9}{9}$. (vi) $2 - \frac{1}{3}$.

(vii) $2 - 1\frac{2}{3}$. (viii) $2 - 1\frac{1}{3}$. (ix) $2 - \frac{2}{7}$.

(x) $5 - 3\frac{3}{7}$. (xi) $\frac{15}{12} - \frac{5}{6}$. (xii) $7\frac{4}{5} - 3\frac{4}{5}$.

2. Fill in the frames :

(i) $1 - \square = \frac{3}{5}$. (ii) $\frac{8}{13} - \square = \frac{5}{13}$.

(iii) $\frac{11}{24} - \square = \frac{1}{3}$. (iv) $\frac{5}{6} - \square = \frac{5}{6}$.

(v) $\frac{3}{4} - \square = 0$. (vi) $\square - \frac{3}{4} = 1$.

3. Examine if each of the following is a true statement :

(i) $\frac{9}{10}-\frac{3}{10}-\frac{1}{10}=\frac{9}{10}-\frac{4}{10}$. (ii) $\frac{9}{10}-\frac{3}{10}-\frac{1}{10}=\frac{8}{10}-\frac{3}{10}$.

(iii) $\frac{7}{9}-\frac{5}{9}-\frac{2}{9}=\frac{7}{9}-\frac{7}{9}$. (iv) $\frac{7}{9}-\frac{5}{9}-\frac{2}{9}=\frac{5}{9}-\frac{5}{9}$.

(v) $\frac{6}{7}-\frac{3}{7}-\frac{2}{7}=\frac{6}{7}-\frac{5}{7}$ (vi) $\frac{6}{7}-\frac{3}{7}-\frac{2}{7}=\frac{4}{7}-\frac{3}{7}$

4. Find the following :

(i) $\frac{7}{8} - \frac{3}{8} - \frac{1}{8}$. (ii) $\frac{5}{4} - \frac{3}{4} - \frac{1}{2}$

(iii) $\frac{5}{6} - \frac{1}{6} + \frac{1}{3}$. (iv) $1\frac{3}{4} - \frac{1}{2} - \frac{3}{4}$.

(v) $2\frac{3}{5} - \frac{4}{5} - \frac{1}{5}$. (vi) $2\frac{3}{5} - \frac{4}{5} + \frac{1}{5}$.

(vii) $2\frac{3}{5} + \frac{4}{5} - \frac{1}{5}$. (viii) $1\frac{3}{7} + \frac{4}{7} - \frac{4}{7}$.

(ix) $2\frac{2}{9} + \frac{2}{3} - \frac{7}{9}$. (x) $1\frac{2}{2} + \frac{1}{7} - \frac{1}{8}$.

Written

5. Find :

(i) $7\frac{5}{6}-3\frac{5}{8}$. (ii) $4\frac{7}{10}-5\frac{11}{12}$. (iii) $\frac{11}{12}-\frac{13}{16}$

(iv) $1\frac{1}{6}-\frac{1}{9}$. (v) $2\frac{3}{4}-1\frac{5}{6}$. (vi) $\frac{17}{6}-\frac{9}{10}$.

6. Find the sum and write down two subtractions for each addition :

(i) $\frac{2}{3}+\frac{5}{6}$, (ii) $\frac{5}{8}+\frac{1}{6}$. (iii) $\frac{7}{10}+\frac{7}{12}$.

(iv) $1+\frac{2}{9}$ (v) $\frac{1}{6}+\frac{1}{9}$. (vi) $\frac{8}{3}+\frac{5}{6}$.

(vii) $\frac{1}{4}+0$. (viii) $0+\frac{3}{11}$ (ix) $\frac{2}{7}+\frac{5}{7}$.

7. For each of the following subtractions give the addition as well as the other subtraction :

(i) $\frac{4}{11}-\frac{3}{11}=\frac{1}{11}$. (ii) $\frac{7}{10}-\frac{1}{6}=\frac{8}{15}$.

(iii) $3\frac{1}{2}-\frac{1}{4}=3\frac{1}{4}$ (iv) $1-\frac{6}{13}=\frac{7}{13}$.

8. Find the following :

(i) $\frac{15}{9}-\frac{2}{3}-\frac{5}{6}$. (ii) $4\frac{5}{8}-1\frac{1}{4}-2\frac{1}{2}$.

(iii) $5-\left(\frac{8}{11}-\frac{3}{11}\right)$. (iv) $\left(5-\frac{8}{11}\right)-\frac{3}{11}$.

(v) $6\frac{1}{2}-\frac{1}{2}-\frac{1}{8}$. (vi) $1\frac{1}{4}-\frac{3}{8}-\frac{1}{8}$.

9. Fill in the frames :

(i) $\frac{5}{8}-\frac{3}{8}+\frac{1}{8}=\square$.   (ii) $\frac{11}{12}-\square-\frac{1}{12}=\frac{2}{3}$.

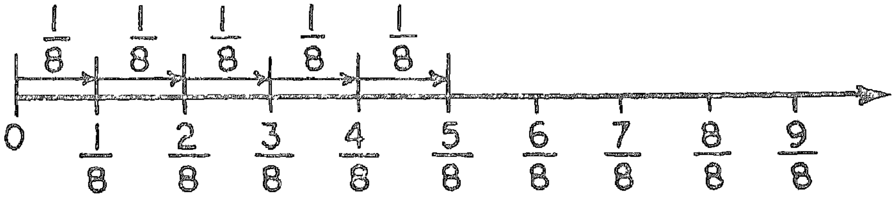
(iii) $\square+3+4\frac{1}{5}=10\frac{4}{5}$ (iv) $\square+3-4\frac{1}{5}=10\frac{4}{5}$.

(v) $\square-3+4\frac{1}{5}=10\frac{4}{5}$.   (vi) $\square-3-4\frac{1}{5}=10\frac{4}{5}$.

3-7 Multiplication in Q_+

In the set *W*, of whole numbers, the multiplication is shown as a repeated addition. It has already been noted that *W* is a subset of Q_+ . Therefore it is natural that multiplication in Q_+ is so defined that whenever two whole numbers are considered as fractional numbers, their product in Q_+ confirms with their product as whole numbers. Let us therefore start with multiplication of a fractional number with a whole number as repeated addition.

Taking the number ray with points associated with fractional numbers having 8 as the common denominator, we find that



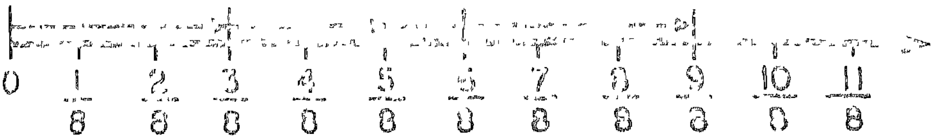
$$\frac{2}{1}\times\frac{1}{8}=2\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}=\frac{2}{8}=\frac{2\times1}{1\times8},$$

$$\frac{3}{1}\times\frac{1}{8}=3\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}=\frac{3\times1}{1\times8},$$

$$\frac{4}{1}\times\frac{1}{8}=4\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{4}{8}=\frac{4\times1}{1\times8},$$

and so on.

Similarly



$$\frac{2}{1} \times \frac{3}{8} = 2 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{2 \times 3}{1 \times 8},$$

$$\frac{3}{1} \times \frac{3}{8} = 3 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{9}{8} = \frac{3 \times 3}{1 \times 8},$$

and so on.

Thus we observe that if we consider multiplication by a whole number as repeated addition, then whenever $\frac{a}{b} \in Q_+$ and $n \in W$, then

$$n \times \frac{a}{b} = \frac{n}{1} \times \frac{a}{b} = \frac{n \times a}{1 \times b}$$

This naturally leads us to define the multiplication of any two fractional numbers as a product obtained by multiplying the numerator by numerator and denominator by denominator, i.e.

$$\frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{15}{24} = \frac{5}{8},$$

$$\frac{2}{3} \times \frac{4}{1} = \frac{2 \times 4}{3 \times 1} = \frac{8}{3},$$

and $$\frac{3}{1} \times \frac{4}{1} = \frac{3 \times 4}{1 \times 1} = \frac{12}{1}.$$

Thus we define that

$$\text{if} \quad \frac{a}{b} \in Q_+, \frac{c}{d} \in Q_+,$$

$$\text{then} \quad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

Example : Find $1\frac{2}{3} \times 4\frac{1}{2}$.

Solution : $1\frac{2}{3} \times 4\frac{1}{2} = \frac{5}{3} \times \frac{9}{2} = \frac{45}{6} = \frac{15}{2} = 7\frac{1}{2}$

EXERCISE SET 3-7

Oral

1. Give the product :

(i) $\frac{1}{2} \times \frac{1}{3}$. (ii) $\frac{2}{3} \times \frac{2}{3}$. (iii) $\frac{2}{3} \times \frac{3}{4}$.

(iv) $3 \times \frac{1}{3}$. (v) $4 \times \frac{1}{8}$. (vi) $\frac{4}{5} \times \frac{5}{4}$.

(vii) $\frac{3}{4} \times \frac{8}{9}$. (viii) $\frac{2}{5} \times \frac{5}{4}$. (ix) $\frac{3}{7} \times \frac{1}{3}$.

(x) $6 \times \frac{5}{6}$. (xi) $6 \times \frac{5}{12}$ (xii) $9 \times \frac{5}{3}$.

Written

2. Find the product and give it in simplest form :

(i) $\frac{8}{12} \times \frac{6}{5}$ (ii) $\frac{3}{2} \times \frac{4}{9}$. (iii) $\frac{5}{4} \times \frac{8}{3}$.

(iv) $6 \times \frac{3}{4}$. (v) $10 \times \frac{3}{4}$. (vi) $\frac{11}{20} \times \frac{24}{11}$.

3 Find the product and give it in simplest form :

(i) $3\frac{1}{2} \times 2\frac{1}{7}$. (ii) $4\frac{1}{2} \times 3\frac{1}{3}$. (iii) $1\frac{1}{2} \times 1\frac{1}{2}$.

(iv) $2\frac{3}{5} \times 3\frac{4}{7}$ (v) $6\frac{3}{4} \times 8$. (vi) $10 \times 1\frac{2}{5}$.

*4 For each product obtained in Ex 2 and Ex. 3, write down two divisions.

*5. Examine if the following statements are true and discover the pattern :

(i) $3 \times 1\frac{1}{2} = 3 + 1\frac{1}{2}.$

$5 \times 1\frac{1}{4} = 5 + 1\frac{1}{4}.$

$9 \times 1\frac{1}{8} = 9 + 1\frac{1}{8}.$

(ii) $3 \times \frac{3}{4} = 3 - \frac{3}{4}.$

$5 \times \frac{5}{6} = 5 - \frac{5}{6}.$

$7 \times \frac{7}{8} = 7 - \frac{7}{8}.$

3-8 Properties of Multiplication in Q_+

Commutative Property

You know that multiplication is commutative in the set of whole numbers.

Consider $4 \times 5.$

We can express 4 and 5 as fractional numbers :

$$4 \times 5 = \frac{12}{3} \times \frac{10}{2}.$$

Since $4 \times 5 = 5 \times 4,$

we find $\frac{12}{3} \times \frac{10}{2} = \frac{10}{2} \times \frac{12}{3}.$

We also see that $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}.$

and $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}.$

Thus $\frac{4}{5} \times \frac{2}{3} = \frac{2}{3} \times \frac{4}{5}$.

Similarly $\frac{3}{3} \times \frac{1}{6} = \frac{1}{6} \times \frac{3}{3}$,

$\frac{7}{4} \times \frac{5}{3} = \frac{5}{3} \times \frac{7}{4}$,

and so on Thus

Multiplication in Q_+ satisfies the commutative property.

In symbols, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two fractional numbers, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}.$$

Associative property

Consider $\frac{2}{3} \times \frac{5}{6} \times \frac{4}{7}$

Let us find the value by grouping them differently, as multiplication is a two-number operation.

$$\left(\frac{2}{3} \times \frac{5}{6}\right) \times \frac{4}{7} = \frac{10}{18} \times \frac{4}{7} = \frac{40}{126},$$

$$\frac{2}{3} \times \left(\frac{5}{6} \times \frac{4}{7}\right) = \frac{2}{3} \times \frac{20}{42} = \frac{40}{126}.$$

So $\left(\frac{2}{3} \times \frac{5}{6}\right) \times \frac{4}{7} = \frac{2}{3} \times \left(\frac{5}{6} \times \frac{4}{7}\right)$.

You can verify by taking more examples. In fact, as multiplication in W is associative.

Multiplication in Q_+ also satisfies the associative property.

In symbols, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three fractional numbers, then

$$\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right).$$

As in addition, the arrangement of terms is very useful in evaluating products of fractional numbers

Examples :

(i) Find $\frac{7}{2} \times \frac{3}{5} \times 4 \times \frac{1}{6}$

Solution : $\frac{7}{2} \times \frac{3}{5} \times 4 \times \frac{1}{6} = \frac{7}{2} \times 4 \times \frac{3}{5} \times \frac{1}{6}$

$$= 14 \times \frac{1}{10}$$

$$= \frac{14}{10} = \frac{7}{5} = 1\frac{2}{5}.$$

(ii) Find $\frac{5}{8} \times \frac{9}{4} \times \frac{8}{15} \times \frac{4}{9}$

Solution : $\frac{5}{8} \times \frac{9}{4} \times \frac{8}{15} \times \frac{4}{9} = \frac{5}{8} \times \frac{8}{15} \times \frac{9}{4} \times \frac{4}{9}$

$$= \frac{1}{3} \times 1$$

$$= \frac{1}{3}.$$

Distributive Property

In W, we have seen the important property of distributivity of multiplication over addition. We shall find if such a property exists in Q_+.

For example let us verify if

$$\frac{3}{4} \times \left(\frac{4}{5}+\frac{6}{7}\right) = \left(\frac{3}{4}\times\frac{4}{5}\right) + \left(\frac{3}{4}\times\frac{6}{7}\right) \text{ is true.}$$

Now $\frac{3}{4} \times \left(\frac{4}{5}+\frac{6}{7}\right) = \frac{3}{4} \times \left(\frac{28}{35}+\frac{30}{35}\right)$

$$= \frac{3}{4} \times \frac{58}{35} = \frac{174}{140}$$

$$\left(\frac{3}{4}\times\frac{4}{5}\right) + \left(\frac{3}{4}\times\frac{6}{7}\right) = \frac{12}{20} + \frac{18}{28}$$

$$= \frac{84}{140}+\frac{90}{140} = \frac{174}{140}$$

So $\frac{3}{4} \times \left(\frac{4}{5}+\frac{6}{7}\right) = \left(\frac{3}{4}\times\frac{4}{5}\right) + \left(\frac{3}{4}\times\frac{6}{7}\right)$ is true.

You can verify it by taking more examples. We shall have *Multiplication in Q_+ is distributive over addition.*

In symbols, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three fractional numbers,

then $\frac{a}{b} \times \left(\frac{c}{d}+\frac{e}{f}\right) = \left(\frac{a}{b}\times\frac{c}{d}\right) + \left(\frac{a}{b}\times\frac{e}{f}\right)$.

We have seen the property of multiplication being distributive over subtraction also in the set of whole numbers. We shall examine it in the set of fractional numbers. For example,

$$\frac{3}{4} \times \left(\frac{6}{7}-\frac{4}{5}\right) = \frac{3}{4} \times \left(\frac{30}{35}-\frac{28}{35}\right)$$

$$= \frac{3}{4} \times \frac{2}{35} = \frac{6}{140}$$

And $\left(\frac{3}{4}\times\frac{6}{7}\right) - \left(\frac{3}{4}\times\frac{4}{5}\right) = \frac{18}{28} - \frac{12}{20}$

$$= \frac{90}{140}-\frac{84}{140} = \frac{6}{140}.$$

So $\frac{3}{4} \times \left(\frac{6}{7}-\frac{4}{5}\right) = \left(\frac{3}{4}\times\frac{6}{7}\right) - \left(\frac{3}{4}\times\frac{4}{5}\right)$.

You can verify by taking more examples.
So Multiplication in Q_+ is distributive over subtraction also.

Multiplication by Zero

We observe that

$$\frac{3}{11} \times 0 = \frac{3}{11} \times \frac{0}{1} = \frac{3 \times 0}{11 \times 1} = \frac{0}{11} = 0.$$

$$5\frac{2}{7} \times 0 = \frac{37}{7} \times \frac{0}{1} = \frac{37 \times 0}{7 \times 1} = \frac{0}{7} = 0$$

Thus it can easily be noted that

Any fractional number multiplied by Zero is equal to Zero.

Multiplicative Identity

In the set of whole numbers W, 1 is multiplicative identity We can easily see that 1 i.e, $\frac{1}{1}$ is the multiplicative identity in the set of fractional numbers also.

We have already seen that

$$\frac{7}{4} \times 1 = \frac{7}{4}\,;\, \frac{5}{9} \times 1 = \frac{5}{9}, \text{ and so on}$$

Any fractional number multiplied by 1 is equal to itself or we say that 1 is the multiplicative identity in Q_+.

Multiplicative Inverse

Does there exist a number in Q_+ whose product with a given number is equal to unity ? This is easily answered. Suppose the number given is $\frac{8}{5}$, we can guess the other number to be $\frac{5}{8}$. We have obviously

$$\frac{8}{5} \times \frac{5}{8} = \frac{8 \times 5}{5 \times 8} = \frac{1}{1} = 1$$

Similarly $\frac{8}{1} \times \frac{1}{8} = \frac{8 \times 1}{1 \times 8} = \frac{8}{8} = 1$

If $\frac{a}{b}$ is any given fractional number different from zero, then the required other number is $\frac{b}{a}$, since their product $\frac{a}{b} \times \frac{b}{a} = \frac{a \times b}{b \times a} = 1$ Here $\frac{a}{b}$ is called the *reciprocal* or more properly the multiplicative inverse of $\frac{b}{a}$. Thus $\frac{a}{b}$ is the multiplicative inverse of $\frac{b}{a}$ and $\frac{b}{a}$ is the multiplicative inverse of $\frac{a}{b}$.

We note that $\frac{a}{b} \times \frac{b}{a} = 1.$

Examples : 1. The multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3}$.

2. The multiplicative inverse of $\left(2\frac{5}{8}\right)$, i. e., of $\frac{21}{8}$ is $\frac{8}{21}$.

3. The multiplicative inverse of $\left(\frac{5}{3}\right)^2$, i.e., of $\frac{5^2}{3^2}$ is evidently equal to $\frac{3^2}{5^2} = \frac{9}{25} = \left(\frac{3}{5}\right)^2$.

4. The multiplicative inverses of natural numbers, say 8, 100, 325, etc. i.e., of $\frac{8}{1}$, $\frac{100}{1}$, $\frac{325}{1}$ are evidently the reciprocals $\frac{1}{8}$, $\frac{1}{100}$, $\frac{1}{325}$, etc.

Notes : 1. To every non-zero fractional number there is a multiplicative inverse such that their product is 1.

2. The reciprocal of the fractional number, zero, does not exist as $\frac{1}{0}$ is not at all a defined number.

3. The reciprocal of 1 is equal to one itself.

EXERCISE SET 3-8

Oral

1 Give the missing number in the following :

(i) $\frac{7}{15} \times \square = 1.$ (ii) $\frac{9}{4} \times \square = 1.$

(iii) $\square \times \frac{1}{7} = 1.$ (iv) $\square \times 16 = 1.$

(v) $\frac{3}{4} \times \square = 2.$ (vi) $\square \times \frac{8}{3} = 3.$

2. Give the reciprocal of the following :

(i) $\frac{1}{7}$. (ii) $\frac{2}{5}$. (iii) $\frac{8}{3}$. (iv) $\frac{1}{8}$. (v) 6. (vi) $\frac{8}{8}$.

3. (i) Which number has no reciprocal in the set of fractional numbers ?

   (ii) Which number in the set of fractional numbers has its reciprocal equal to itself ?

   (iii) What is the product of a number and its reciprocal?

Written

4. Give the product :

(i) $3\frac{1}{2} \times 4$. (ii) $5 \times 1\frac{3}{5}$. (iii) $10 \times 2\frac{2}{5}$.

(iv) $8 \times 4\frac{3}{4}$. (v) $8 \times 1\frac{7}{8}$. (vi) $\frac{1}{2} \times 2\frac{2}{3}$.

(vii) $\frac{1}{4} \times 8\frac{4}{5}$. (viii) $\frac{1}{3} \times 9\frac{3}{4}$.

5. Simplify :

(i) $3\frac{1}{2}\left(\frac{1}{7} + 4\frac{1}{4}\right)$. (ii) $2\frac{2}{3}\left(1\frac{1}{8} - \frac{5}{6}\right)$.

(iii) $\left(5\frac{1}{2} + \frac{3}{11}\right) \times \frac{1}{10}$. (iv) $\left(1\frac{3}{5} - 3\frac{1}{3}\right) \times \frac{3}{8}$.

6. Find the product :

(i) $\frac{2}{3} \times \frac{4}{7} \times \frac{10}{11}$. (ii) $\frac{3}{5} \times \frac{5}{8} \times \frac{8}{3}$.

(iii) $6 \times \frac{1}{6} \times \frac{2}{2}$. (iv) $\frac{7}{2} \times \frac{4}{9} \times 0$.

(v) $\frac{10}{9} \times \frac{3}{5} \times \frac{6}{2}$. (vi) $\frac{17}{8} \times \frac{8}{7} \times \frac{7}{8}$.

7. Find the product ·

(i) $1\frac{1}{4} \times 2\frac{1}{5} \times 2\frac{2}{3}$. (ii) $3\frac{3}{4} \times 4\frac{2}{3} \times 1\frac{1}{7}$.

(iii) $6\frac{1}{7} \times 0 \times 1\frac{1}{2}$. (iv) $2\frac{5}{8} \times 3\frac{1}{7} \times 1\frac{1}{11}$.

8. Use distributive property to simplify the following in simplest form .

(i) $\frac{2}{3}\times\frac{7}{9}+\frac{2}{3}\times\frac{2}{9}$. (ii) $\frac{15}{8} \times \frac{9}{10} - \frac{15}{8} \times \frac{1}{10}$.

(iii) $2\frac{7}{8}\times 3\frac{3}{4}+ 2\frac{7}{8}\times 4\frac{1}{4}$.

(iv) $19 \times 7\frac{1}{4} - 19 \times 6\frac{1}{4}$.

9. Fill in the frames to get true statements. State the general property :

(i) $\frac{3}{4} < \frac{7}{8} \rightarrow \frac{3}{4} \times \square < \frac{7}{8} \times \square$;

(ii) $\frac{9}{5} < \frac{4}{3} \rightarrow \frac{9}{5} \times \square < \frac{4}{3} \times \square$.

(iii) $1\frac{1}{2} > 1\frac{3}{8} \rightarrow 1\frac{1}{2} \times \square > 1\frac{3}{8} \times \square$

(iv) $2\frac{5}{6} > 1\frac{8}{9} \rightarrow 2\frac{5}{6} \times \square > 1\frac{8}{9} \times \square$.

10. Write down the reciprocals of the following :

(i) $\frac{7}{11}$. (ii) $\frac{5}{123}$. (iii) $5\frac{2}{7}$.

(iv) $7\frac{9}{11}$. (v) $13\frac{3}{7}$. (vi) $5\frac{2}{7} \times \frac{5}{111}$.

(vii) $\left(\frac{13}{7}\right)^2$. (viii) $\left(\frac{6}{5}\right)^3$. (ix) $\left(\frac{3}{4}\right)^2 \times \left(\frac{1}{2}\right)^4$.

(x) 121×4. (xi) 0. (xii) 1.

*11. The reciprocal of the product of two fractional numbers is equal to the product of their reciprocals. Take two particular examples and verify the truth of this important result.

3-9 Exponents

We have seen a short way of representing a product of equal whole numbers, using exponents. We shall see how we can extend it to the product of equal fractional numbers. For example,

We write $\frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^2$.

But $\frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3} = \frac{(2)^2}{(3)^2}$.

So $\left(\frac{2}{3}\right)^2 = \frac{(2)^2}{(3)^2}$.

Again $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^3$.

But $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{(2)^3}{(3)^3}$.

So $\left(\frac{2}{3}\right)^3 = \frac{(2)^3}{(3)^3}$.

In general, if $\frac{a}{b}$ is a fractional number, then

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}.$$

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}.$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

Example : Solve $\left(\frac{3}{4}\right)^2 \left[\left(\frac{5}{8}\right)^2 - \left(\frac{1}{3}\right)^2 \right]$.

Solution : Given expression

$$= \frac{9}{16} \left[\frac{25}{64} - \frac{1}{9} \right].$$

$$= \frac{9}{16} \left[\frac{225 - 64}{9 \times 64} \right].$$

$$= \frac{9 \times 161}{16 \times 9 \times 64}.$$

$$= \frac{161}{1024}.$$

EXERCISE SET 3-9

Oral

1. Simplify :

(i) $\left(\frac{1}{2}\right)^3$. (ii) $\left(\frac{1}{4}\right)^2$. (iii) $\left(\frac{3}{4}\right)^2$. (iv) $\left(\frac{5}{2}\right)^3$.

2. Simplify :

(i) $\frac{10^5}{2^5}$. (ii) $\frac{(36)^3}{(12)^3}$. (iii) $\frac{(16)^2}{(12)^2}$. (iv) $\frac{(6)^5}{(2)^5}$.

Written

3. Simplify :

(i) $\left(\frac{2}{3}\right)^3\left[\left(\frac{7}{8}\right)^2+\left(\frac{1}{4}\right)^2\right]$.

(ii) $\left(\frac{1}{2}\right)^3+\left(\frac{2}{3}\right)^3+\left(\frac{5}{6}\right)^3$.

(iii) $\left(\frac{3}{4}\right)^3+\left(\frac{5}{6}\right)^3-\left(\frac{1}{12}\right)^3$.

3-10 Division in Q_+

As in the set of whole numbers, we can write two divisions for every multiplication in the set of fractional numbers. For example, let the multiplication be

$$\frac{3}{4}\times\frac{5}{7}=\frac{15}{28}, \qquad \text{(i)}$$

so that the two divisions are

$$\frac{15}{28}\div\frac{3}{4}=\frac{5}{7} \text{ and } \frac{15}{28}\div\frac{5}{7}=\frac{3}{4}. \qquad \text{(ii)}$$

Multiplying both sides of (i) by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$, we have

$$\frac{4}{3}\times\frac{3}{4}\times\frac{5}{7}=\frac{4}{3}\times\frac{15}{28},$$

or $$\left(\frac{4}{3}\times\frac{3}{4}\right)\times\frac{5}{7}=\frac{15}{28}\times\frac{4}{3},$$

or $$1\times\frac{5}{7}=\frac{15}{28}\times\frac{4}{3},$$

or $$\frac{5}{7}=\frac{15}{28}\times\frac{4}{3} \qquad \text{(iii)}$$

We thus see that

$$\frac{5}{7} = \frac{15}{28} - \frac{3}{4} \text{ and } \frac{5}{7} = \frac{15}{28} \times \frac{4}{3}.$$

So
$$\frac{15}{28} - \frac{3}{4} = \frac{15}{28} \times \frac{4}{3}.$$

Again multiplying both sides of (i) by $\frac{7}{5}$, the reciprocal of $\frac{5}{7}$, we have

$$\frac{3}{4} \times \frac{5}{7} \times \frac{7}{5} = \frac{15}{28} \times \frac{7}{5}.$$

or
$$\frac{3}{4} \times \left(\frac{5}{7} \times \frac{7}{5}\right) = \frac{15}{28} \times \frac{7}{5}.$$

or
$$\frac{3}{4} \times 1 = \frac{15}{28} \times \frac{7}{5}.$$

We thus see that

$$\frac{3}{4} = \frac{15}{28} - \frac{5}{7} \text{ and } \frac{3}{4} = \frac{15}{28} \times \frac{7}{5}.$$

So
$$\frac{15}{28} \div \frac{5}{7} = \frac{15}{28} \times \frac{7}{5}.$$

From these examples we can easily see that :

Dividing a given fractional number by another fractional number is equivalent to multiplying the given fractional number by the reciprocal of the dividing fractional number.

In symbols if $\frac{a}{b}$ and $\frac{c}{d}$ are any two fractional numbers, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Again $\frac{6}{20} \div \frac{3}{4}$ can also be written as $\dfrac{\frac{6}{20}}{\frac{3}{4}}$.

This is called a complete fraction as the numerator and the denominator are fractions.

Now $$\frac{\frac{6}{20}}{\frac{3}{4}} = \frac{\frac{6}{20} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{\frac{6}{20} \times \frac{4}{3}}{1} = \frac{6}{20} \times \frac{4}{3}.$$

$$\therefore \frac{6}{20} \div \frac{3}{4} = \frac{6}{20} \times \frac{4}{3}.$$

Example 1 : Find $1 \div \frac{7}{9}$.

Solution : $1 \div \frac{7}{9} = 1 \times \frac{9}{7} = \frac{9}{7}$.

Example 2 : Find $2\frac{4}{5} \div 1\frac{3}{4}$.

Solution : $$2\frac{4}{5} \div 1\frac{3}{4} = \frac{14}{5} \div \frac{7}{4}.$$
$$= \frac{14}{5} \times \frac{4}{7}.$$
$$= \frac{14 \times 4}{5 \times 7}. = \frac{8}{5}.$$

Example 3 : Find $0 \div \frac{5}{2}$.

Solution : $0 \div \frac{5}{2} = 0 \times \frac{2}{5} = 0$.

Example 4 : Find $\frac{\frac{3}{4}}{2}$

Solution : $$\frac{\frac{3}{4}}{2} = \frac{\frac{3}{4} \times \frac{1}{2}}{2 \times \frac{1}{2}} = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}.$$

Note :—Dividing a fraction by 2 is the same as multiplying the fraction by $\frac{1}{2}$.

EXERCISE SET 3-10

Oral

1. State the following divisions as multiplications :

(i) $\frac{7}{3} \div \frac{2}{5}$. (ii) $\frac{1}{2} \div \frac{1}{8}$. (iii) $\frac{8}{5} \div 4$.

(iv) $\frac{3}{10} \div 5$. (v) $2 \div \frac{1}{3}$. (vi) $\frac{4}{7} \div \frac{1}{7}$.

2. Find the following :

(i) $\frac{1}{4} \div \frac{1}{2}$. (ii) $\frac{1}{2} \div \frac{1}{4}$. (iii) $\frac{3}{2} \div \frac{2}{3}$.

(iv) $\frac{3}{2} \div \frac{2}{3}$. (v) $\frac{3}{4} \div 3$. (vi) $8 \div \frac{3}{4}$.

(vii) $\frac{4}{9} \div \frac{1}{3}$. (viii) $\frac{10}{11} \div 10$. (ix) $15 \div \frac{3}{2}$.

Written

3. Find :

(i) $\frac{8}{15} \div \frac{2}{3}$. (ii) $\frac{3}{4} \div \frac{9}{20}$,

(iii) $\frac{10}{9} \div \frac{5}{9}$. (iv) $\frac{8}{11} \div \frac{16}{77}$.

(v) $\frac{35}{33} \div \frac{7}{11}$. (vi) $\frac{9}{2} \div \frac{99}{20}$.

*4. Find the pattern :

(i) $\frac{4}{5} = \frac{\frac{1}{5}}{\frac{1}{4}}$ (ii) $4 \times 7 = \frac{4}{\frac{1}{7}}$.

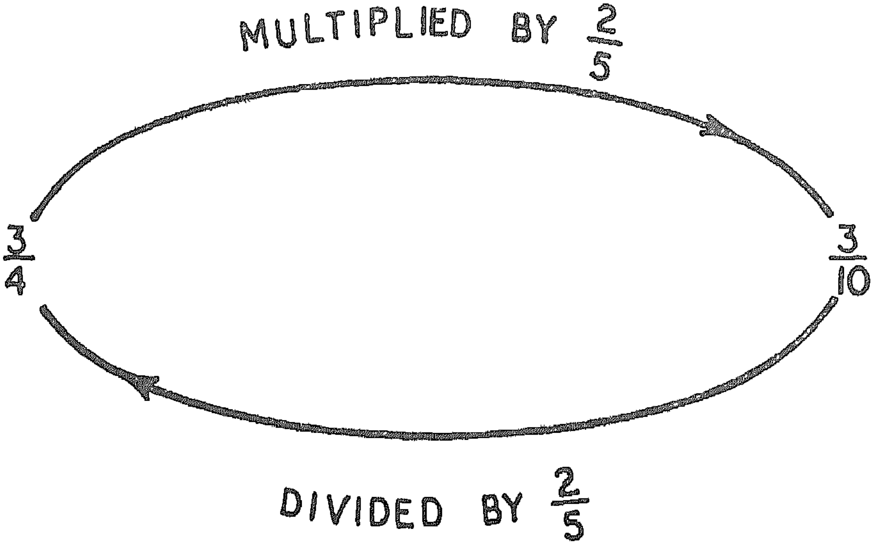
$\frac{3}{2} = \frac{\frac{1}{2}}{\frac{1}{3}}$ $3 \times 6 = \frac{3}{\frac{1}{6}}$.

$\frac{7}{9} = \frac{\frac{1}{9}}{\frac{1}{7}}$ $9 \times 8 = \frac{9}{\frac{1}{8}}$.

Give an example of the above type.

3-11 Properties of Division in Q_+

Division is the inverse of multiplication (in a certain sense) as is clear from the figure given below :



Closure Property :—In the set of whole numbers, division of one whole number by another does not yield always a whole number. But in the set of fractional numbers, division of one fractional number by anothor non-zero fractional number yields *always* a fractional number as we have already seen through many examples in section (3–10).

Thus the set of fractional numbers from which 0 is removed is closed with respect to the division operation.

EXERCISE SET 3-11

Oral

1. Fill in the frames :

(i) $\frac{7}{2} \div \frac{3}{4} = \square$. (ii) $\frac{11}{4} \div \square = \frac{8}{5}$.

(iii) $\square \div \frac{8}{3} = \frac{7}{4}$. (iv) $1 \div \frac{5}{7} = \square$.

(v) $1 \div \square = \frac{4}{11}$. (vi) $\square \div \frac{5}{12} = \frac{12}{5}$.

(vii) $0 \div \frac{1}{2} = \square$. (viii) $\square \div \frac{7}{8} = 0$.

Written

2. Find the following :

(i) $4\frac{1}{2} \div 2\frac{1}{4}$. (ii) $\left(6\frac{2}{5} \div 3\frac{1}{5}\right) \div 1\frac{1}{2}$.

(iii) $6\frac{2}{5} \div \left(3\frac{1}{5} \div 1\frac{1}{2}\right)$. (iv) $2\frac{1}{3} \div \left(5\frac{1}{2} + 3\frac{3}{4}\right)$.

(v) $\left(\frac{7}{8} - \frac{3}{5}\right) \div \frac{1}{2}$. (vi) $\frac{1}{2} \div \left(\frac{7}{8} - \frac{3}{5}\right)$.

3. Each of the following is one of the division for a multiplication. Give the other division :

(i) $\frac{5}{8} \div \frac{10}{3} = \frac{3}{16}$ (ii) $1 \div \frac{1}{3} = 3$.

*4. Find the following :

(i) $\left(2\frac{1}{2} \times 1\frac{1}{5}\right) \div \frac{1}{6}$.

(ii) $\left(3\frac{3}{4} \div 2\frac{2}{3}\right) \times 3\frac{1}{5}$.

(iii) $\frac{5}{9} \div \left(\frac{10}{3} \div \frac{12}{5}\right)$.

*5. Discover the pattern :

(i) $5\frac{1}{7} \div \frac{6}{7} = 5\frac{1}{7} + \frac{6}{7}$. (ii) $5\frac{1}{3} \div 4 = 5\frac{1}{3} - 4$.

$6\frac{1}{8} \div \frac{7}{8} = 6\frac{1}{8} + \frac{7}{8}$. $6\frac{1}{4} \div 5 = 6\frac{1}{4} - 5$.

$3\frac{1}{5} \div \frac{4}{5} = 3\frac{1}{5} + \frac{4}{5}$. $9\frac{1}{7} - 8 = 9\frac{1}{7} - 8$.

Give one more example of the above type.

3-12 Fractions as Ratios

Consider :

(1) two vessels containing water which (by using a sufficiently small unit of measure, say, cubic centimeter) contain 1043 units and 3045 units, respectively;

(2) two lumps of sugar weighing 345 and 451 grams respectively,

(3) two pieces of cloth measuring 3 and 2 metres long respectively;

(4) two line segments which are 47 and 53 cm in length respectively;

These situations can be expressed using a new expression called ratio (whose origin goes back to the times, when *fraction* with its modern notation was not at all in universal use).

We say that

(1) the ratio of the water-contents by volume in the two vessels is as 1043 is to 3045, written as 1043 : 3045 ;

(2) the ratio of the weights of the two lumps of sugar is 345 : 451;

(3) the ratio of the lengths of two pieces of cloth is 3 : 2;

(4) the ratio of the lengths of the line segments is 47 : 53.

What we mean by the symbol $a : b$ is really the fraction $\frac{a}{b}$.

Observe that we are measuring at each time the same kind of quantity. Suppose we have two ratios a : b and c : d, we say that the ratios are equal only when:

$$\frac{a}{b} = \frac{c}{d}.$$

This is also expressed by

$a : b : : c : d$ (read as a to b is such as c to d)

Or by $a : b = c : d$ (read as a to b is equal to c to d),

Note :—The fraction with the convenient modern notation be always employed.

Example : Divide a line-segment 84 cm in length in the ratio of 3 : 4.

Solution : What is implied is that the ratio of the two parts by length is $\frac{3}{4}$. The lengths of these parts must therefore be $3x$ and $4x$ respectively.

where $x \in Q_+$

Now $3x + 4x$=Total length $= 84$ (in cm)

$\rightarrow 7x = 84 \rightarrow x = 12$

$\rightarrow$ the length of the two segments are $3 \times 12 = 36$ cm, and $4 \times 12 = 48$ cm respectively.

Rate : Very often we ask a question as follows :

If 4 kilos cost Rs. 5, what is the price of 1 kilo ?

The price of 1 kilo is Rs. 5/4, or the price is *Rs. 5/4 per kilo.*

Here the fractional number is used as a *rate.*

Consider another example :

If 500 apples are distributed equally among 20 persons, how much does each get ?

The question can also be reworded as :

At what rate is the distribution made if 500 apples are distributed equally among 20 persons ?

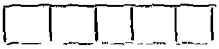
The rate of distribution is $\frac{500}{20}$ or 25 apples per head.

EXERCISE SET 3-12

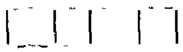
Oral

1. Compare the first region with the second in the following figures and give your answer in the form of a fraction as well as a ratio :

(i)

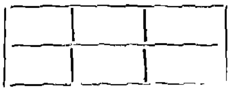


Region M

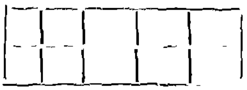


Region N

(ii)

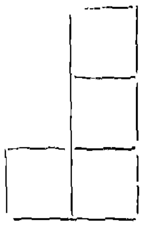


Region K

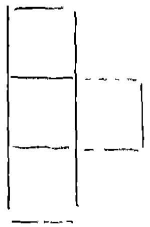


Region L

(iii)



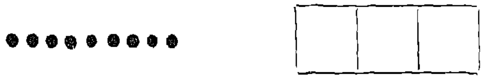
Region E



Region F

2. Give the rate in the following cases :

   (i) 100 pencils for Rs. 42.
   (ii) 20 apples distributed equally among 5 persons.
   (iii) 200 packets distributed equally among 5 boxes.
   (iv) 400 litres of milk shared equally by 80 persons.
   (v) The set of dots given below distributed equally in the squares



3. Complete following :

   (i) A has Rs. 2,000 and B Rs. 3,000. What A has is...... of what B has.
   (ii) A has 500 kilos of sugar and B 200 kilos of sugar. What A has is............of what B has.
   (iii) A has 80 sheep and B 25 sheep. What A has is ....of what B has.
   (iv) A has 360 litres of paint and B 40 litres of paint. What B has is............of what A has.

Written

4. Fill in the frames :

   (i) $15 : 20 = \square : 12$
   (ii) $\square : 30 = 35 : 42$.

*5. Two persons A and B share a sum of money. Complete the following :

   (i) If A gets $\frac{1}{5}$ more than what B gets, B gets......... less than what A gets.
   (ii) If A gets $\frac{1}{6}$ more than what B gets, B gets......... less than what A gets

(iii) If A gets $\frac{1}{7}$ more than what B gets, B gets........... less than what A gets.

Discover the pattern.

*6. Two persons A and B share a quantity. Complete the following :

(i) If A gets $\frac{2}{5}$ more than what B gets, B gets........... less than what A gets.

(ii) If A takes $\frac{3}{8}$ more than what B gets, B gets......... less than what A gets.

(iii) If A takes $\frac{5}{7}$ more than what B gets, B gets......... less than what A gets.

3-13 Fractional Numbers in other forms

Decimals

In our base ten numeration system, we group successively by tens and then form places.

Place values chart

...	10000	1000	100	10	1

Consider the place value chart. As we proceed from right to left, each place to the left is ten times the place value to its right We shall now consider place values from the left to the right.

1000 is 1/10 of 10000

100 is 1/10 of 1000

10 is 1/10 of 100

1 is 1/10 of 10.

That is to say, *each place value is 1/10 of the place value to its left.*

We shall take decimal numeration beyond the unit. To indicate this extension, a point or comma is used as shown below :

Consider 4238·7.

Now what is the value of the place at 7 ?
It is 1/10 of the value of the place at 8. So the place value of 7 is 7/10 (seven tenths).

We can write 4238·7 in the expanded notation.

$$4238{\cdot}7 = 4 \times 1000 + 2 \times 100 + 3 \times 10 + 8 \times 1 + \frac{7 \times 1}{10}.$$

Suppose we annex another digit to the right of 7 and write

4238 76.

What is the value of the place at 6 ?

It is $\frac{1}{10}$ of the value of the place at 7.

or $\frac{1}{10} \times \frac{1}{10}$ of the value of the place at 8.

So $4238{\cdot}76 = 4 \times 1000 + 2 \times 100 + 3 \times 10 + 8 \times 1 + \frac{7 \times 1}{10} + \frac{6 \times 1}{100}$.

Using exponents, we can write

$$4238{\cdot}76 = 4 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 8 \times 1 + \frac{7}{10} + \frac{6}{10^2}.$$

Now the extended place value chart is as follows :

...	10000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	...
..	10^4	10^3	10^2	10^1	10^0	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	...

Here in 4238 76, 4238 is called the whole part while 76 called the fractional part.

The point separating the whole part from the fractional part of a number is called the decimal point. The places to the right of the decimal point are called decimal places.

One should be careful in reading decimal fraction. 34.732 is read as thirty four point seven, three, two which means thirty four ones, seven tenths, three hundredths and two thousandths.

106·0634 is read as one hundred six point zero, six, three, four.

We can express decimal fractions as simple fractional numbers.

Examples .

$$\text{(i) } 3{\cdot}04 = 3 + \frac{0}{10} + \frac{4}{100} = 3 + \frac{4}{100} = \frac{304}{100}.$$

$$\text{(ii) } 62{\cdot}35 = 6 \times 10 + 2 \times 1 + \frac{3 \times 1}{10} + \frac{5 \times 1}{100}.$$

$$= \frac{6000}{100} + \frac{200}{100} + \frac{30}{100} + \frac{5}{100}.$$

$$= \frac{6235}{100}.$$

Similarly we can express fractional numbers in decimal form :

Examples : $$\text{(i) } \frac{424}{1000} = \frac{400}{1000} + \frac{20}{1000} + \frac{4}{1000}.$$

$$= \frac{4}{10} + \frac{2}{100} + \frac{4}{1000}.$$

$$= 0{\cdot}424.$$

$$\text{(ii) } \frac{3028}{100} = \frac{3000}{100} + \frac{20}{100} + \frac{8}{100}.$$

$$= 30 + \frac{2}{10} + \frac{8}{100}.$$

$$= 30{\cdot}28.$$

$$\text{(iii) } \frac{3}{5} = \frac{6}{10} = 0.6.$$

$$\text{(iv) } \frac{3}{8} = \frac{15}{40} = \frac{75}{200} = \frac{375}{1000} = 0\,375$$

Percentages

In day to day transactions we use a word called **percentage** denoted by %; e.g., we say that the first rank boy scored 82 percent (written as 82%) of the total marks. the maximum of which is 400, say. This is equivalent to saying that his total marks were 328 out of 400. We give another example :

Suppose the height of water in a certain dam on a particular day was 800 metres, and it increased by $1\frac{3}{8}\%$ next day; this means that the height of water increased by $\frac{1\frac{3}{8}}{100}$ of its previous height, i.e., it increased by $\frac{11}{800}$ of 800 or $\frac{11}{800} \times 800 = 11$ metres.

It may be pointed out that such notations were in use before the universal adoptation of the modern notation for a fraction and we can dispense with them in mathematics now but for the fact that they are current in day to day transactions.

Now we have

$$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100}.$$

$$\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100}.$$

A fractional number with 100 in the denominator is called a percentage and it is written usually as shown below :

$$\frac{40}{100} = 40\%.$$

$$\frac{25}{100} = 25\%.$$

$$\frac{50}{100} = 50\%.$$

Percentages are fractional numbers with the same denominator 100. So if two fractional numbers are expressed as percentages, then it is easy to compare them. For example,

Compare $\frac{2}{5}$ and $\frac{7}{20}$.

Now $\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\%.$

$\frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = 35\%,$

Comparing $\frac{2}{5}$ and $\frac{7}{20}$, is the same as comparing the equivalent fractions (now percentages) 40%, and 35%.

Since $40\% > 35\%,$

$$\frac{2}{5} > \frac{7}{20}.$$

We can also express fractions into percentages in another way.

What is 100% ?

$$100\% = \frac{100}{100} = 1.$$

So multiplying a fractional number by 100% is nothing but multiplying the fractional number by the *multiplicative identity*.

So $\frac{2}{5} = \frac{2}{5} \times 100\% = \frac{200}{5}\% = 40\%$

$\frac{1}{4} = \frac{1}{4} \times 100\% = \frac{100}{4}\% = 25\%.$

Similarly $\frac{1}{2} = \frac{1}{2} \times 100\% = \frac{100}{2}\% = 50\%.$

Given a percentage, we can easily express it as a fraction in simplest form,

Examples : (i) $60\% = \frac{60}{100} = \frac{3 \times 20}{5 \times 20} = \frac{3}{5}$.

(ii) $75\% = \frac{75}{100} = \frac{3 \times 25}{4 \times 25} = \frac{3}{4}$.

We can also express percentage as a decimal easily, for example

$$45\% = \frac{45}{100} = \frac{40}{100} + \frac{5}{100} = \frac{4}{10} + \frac{5}{100} = 0{\cdot}45.$$

Applications

Example : *1.* In the first class test, Saxena scored 6 out of 20 and in the second 7 out of 25. In which test did he score more ?

Solution : In the first test he got $\frac{6}{20}$ or $\frac{6}{20} \times 100\%$.

$$= 6 \times \frac{1}{20} \times 100\%.$$

$$= 6 \times 5\% = 30\%$$

In the second test he got $\frac{7}{25}$ or $\frac{7}{25} \times 100\%$.

$$= 7 \times \frac{1}{25} \times 100\%.$$

$$= 28\%.$$

So he scored more marks in the first.

Example : 2. A boy's weight last year was 50 kilos This year it increased by 5%. Find his weight this year.

Solution : Boy's weight in kilos last year = 50 kilos.
Increase in weight this year 5%.

That is $\frac{50 \times 5}{100} = 2{\cdot}5$.

So his weight this year is $(50 + 2{\cdot}5) = 52\ 5$ kilos.

EXERCISE SET 3-13

Oral

1. Read the following :

   (i) 48·976. (ii) 7·90314. (iii) 80·008. (iv) 0·102.

2. Give the place value of the digits **printed in bold** :

   (i) **27**78·**43** (ii) 301·0**4**9**3**. (iii) 0·800**6**2.

3. Express the following as percentages :

   (i) $\frac{4}{5}$. (ii) $\frac{7}{20}$. (iii) $\frac{11}{25}$. (iv) $\frac{3}{2}$. (v) $\frac{5}{4}$.

   (vi) $\frac{1}{200}$. (vii) 2.

4. Express the following percentages as simple fractions :
   (i) 12%. (ii) 28%. (iii) 90%. (iv) 20%.
   (v) 5%. (vi) 10%. (vii) 1%.

5. Express the following as simple fractions :
   (i) 2·8 (ii) 0·28 (iii) 1·44 (iv) 14·4 (v) 0·144 (vi) 1·125.

6. Express the following as decimals :
   (i) 32%. (ii) 4%. (iii) 40%. (iv) 50%. (v) 5%.

7. Express the following as percentages :
   (i) 0·7 (ii) 0·06 (iii) 0·15 (iv) 0·8 (v) 0·02 (vi) 0·72

Written

8. Rewrite the following in numerals :

   (i) Seven hundred sixty one point four, four.

   (ii) Ninety point zero, zero, nine, five.

   (iii) Five ones sixteen tenths, three thousandths.

   (iv) Eight thousand fifty ones three hundredths one thousandth.

9. Rewrite the following in the expanded notation :

   (i) 23·713. (ii) 816·33 (iii) 8·1633.

   (iv) ·07. (v) ·07845. (vi) ·0406.

10. Rewrite the following as decimal fractions :

    (i) $5 \times 100 + 1 \times 10 + 3 \times 1 + 2 \times \left(\frac{1}{10}\right) + 7 \times \left(\frac{1}{100}\right)$.

    (ii) $3 \times 10 + 5 \times 1 + 0 \times \left(\frac{1}{10}\right) + 8 \times \left(\frac{1}{100}\right) + 6 \times \left(\frac{1}{1000}\right)$.

11. Express the following as percentages :

    (i) $\frac{3}{8}$. (ii) $\frac{5}{8}$. (iii) $\frac{1}{3}$. (iv) $\frac{2}{3}$. (v) $\frac{1}{16}$.

12. Express the following as simple fractions :

    (i) $6\frac{2}{3}\%$ (ii) $87\frac{1}{2}\%$. (iii) $16\frac{2}{3}\%$. (iv) $9\frac{1}{11}\%$.

    (v) $11\frac{1}{9}\%$.

13. Express the following as decimal fractions :

    (i) $2 \times 1 + 5 \times \frac{1}{100}$. (ii) $\frac{4}{10} + \frac{7}{1000}$.

    (iii) $\frac{4}{100} + \frac{7}{1000}$. (iv) $\frac{9}{1000}$.

(v) $8 \times 100 + \frac{7}{100}$. (vi) $3 \times 1000 + \frac{2}{1000}$.

(vii) $4 \times 100 + 7 \times 1 + \frac{6}{100}$.

(viii) $6 \times 1000 + 6 \times 10 + \frac{6}{100} + \frac{6}{10000}$.

14. Express the following as decimal fractions of a metre :
    - (i) 5 metres 23 centimetres.
    - (ii) 8 decimetres 4 centimetres.
    - (iii) 24 centimetres 8 millimetres

15. Do the following problems by using percentages :
    - (i) 5/6 of a sweet is flour. In another sweet 11/12 of it is flour. Which sweet has greater amount of flour ?
    - (ii) 3/8 of an alloy is gold and the rest copper. 4/9 of another alloy is copper and the rest gold. Which alloy contains more gold ? Which alloy contains more copper ?

16. (i) A tin contained 5 litres of kerosene. After a week, it was found that there was $4\frac{3}{4}$ litres; the rest had evaporated. What percentage of kerosene evaporated in the week ?
    - (ii) John gets a salary of Rs. 600 per month. His salary will be increased by 5% next year. What will be his monthly salary next year ?

3-14 Decimal Fractions

Addition and Subtraction

Rules for addition and subtraction of dacimals are similar to the rules for addition and subtraction of whole number. For example, in the case of whole numbers if 4532 and 43 are to be added, we write them, one below the other, so that their place values mach, and then add, as shown below ·

$$\begin{array}{r} 4532 \\ +\quad 43 \\ \hline 4575 \end{array}$$

Now suppose we have to add 0·4532 and and 1·63.
We can adopt the same procedure in writing them, one below the other, such that their place values match, and then add, as shown below :

$$\begin{array}{r} 0{\cdot}4532 \\ +\ 1{\cdot}63 \\ \hline 2{\cdot}0832 \end{array}$$

For subtraction the procedure is the same as shown below :

$$\begin{array}{r} 4532 \\ -\quad 43 \\ \hline 4489 \end{array} \qquad \begin{array}{r} 0\ 4532 \\ -0{\cdot}43 \\ \hline 0{\cdot}0232 \end{array} \qquad \begin{array}{r} 3\ 0059 \\ -1{\cdot}8236 \\ \hline 1{\cdot}1823 \end{array}$$

Addition and subtraction of decimals can be explained by writing them as simple fractions.

For example, (i) $0\ 4532 + 0\ 43 = \frac{4532}{10000} + \frac{43}{100}$.

$$= \frac{4532}{10000} + \frac{4300}{10000}.$$

$$= \frac{8832}{10000} = 0{\cdot}8832.$$

(ii) $0{\cdot}4532 - 0{\cdot}43 = \frac{4532}{10000} - \frac{43}{100}$

$$= \frac{4532}{10000} - \frac{4300}{10000}.$$

$$= \frac{232}{10000} = 0{\cdot}0232.$$

By annexing zeros to the right of a decimal, will the value of the decimal get changed ? Let us find it out.

3·70 means $\frac{370}{100}$ which is equivalent to $\frac{37}{10}$ or 3·7.

3·700 mens $\frac{3700}{1000}$ which is equivalent to $\frac{37}{10}$ or 3·7.

3·7000 means $\frac{37000}{10000}$ which is equivalent to $\frac{37}{10}$ or 3·7.

From these, we observe that by annexing any number of zeros to the right of a decimal, the value of decimal remains unchanged.

Ordering

Given any two decimal fractions we can easily find out which of them is greater.

Examples :

1. Consider two decimal fractions 0·8 and 0·791.
   0·8 has 8 tenths, whereas

   0·791 has 7 tenths, 9 hundredths and 1 thousandth
   So 0·8 > 0·791.

2. Consider two decimal fractions 0·062 and 0·0083
   0·062 has 6 hundredths and 2 thousandths.

   0·0083 has 8 thousandths and 3 tenthousandths.
   So 0 062 > 0083.

EXERCISE SET 3-14

Oral

1. Evaluate

   (i) 0 8 + 0·2. (ii) 0·8 + 0·02. (iii) 0·08 + 0·2

   (iv) 0·08 + 0·02. (v) 0·08 + 0·12. (vi) 0·18 + 0·12.

2. Evaluate

(i) 0·7 – 0 5. (ii) 0·7 – 0·05. (iii) 7 – 0·5.

(iv) 7 – 0·05. (v) 1 – 0·8. (vi) 1 – 0·08.

3. Pick out the greater decimal fractions from each of the following pairs :

(i) 0·3, 0·04. (ii) 0·03, 0·04. (iii) 0·006, 0·01.

(iv) 0·71, 0·071. (v) 9·8, ·98 (vi) 3·55, 35·5.

Written

4 Write the following, one below the other, so that their places agree and then find their sum .

(i) 4·5, 6·82. (ii) 0·45, 6·82. (iii) 0·45, 68·2.

(iv) 0·45, 682·0. (v) ·036, ·0036. (vi) 150, ·05.

5. Rewrite the decimals in exercise 4, so as to have equal number of places in the decimal part by annexing zeros wherever necessary :

6. Find the sum :

(i) 34·8 + 2·49. (ii) 7·641 + 0·76.

(iii) 0·045 + ·0006 (iv) 38·002 + 200.

(v) 4·998 + 0·0025. (vi) 7·0909 + 3·909.

(vii) 8·1817 + 0·0203. (viii) 0·0996 + 0 01+0·101.

(ix) 3·126 + 0·3126 + 31·26.

(x) 0·493 + 0·0493 + 0·00493.

6. Find :

(i) 38·51 – 2·64. (ii) 38 – 2·64.

(iii) 3 – 2·04. (iv) 1 – 0·0304.

(v) 1 – 0·0304. (vi) 1 – 0·0017.

(vii) 2·44 – 0·244 (viii) 0·406 – 0·0406.

3·15 Decimal Fractions

Multiplication and Division :

We shall begin with the discovery of patterns seen in multiplying and dividing a decimal by powers of 10.

Let us find the following ·

$$2{\cdot}8314 \times 10$$

$$2{\cdot}8314 \times 100$$

$$2{\cdot}8314 \times 1000$$

Now $2{\cdot}8314 \times 10 = \frac{28314}{10000} \times 10$

$$= \frac{28314}{1000}$$

$$= \frac{20000}{1000} + \frac{8000}{1000} + \frac{300}{1000} + \frac{10}{1000} + \frac{4}{1000}$$

$$= 20 + 8 + \frac{3}{10} + \frac{1}{100} + \frac{4}{1000}$$

$$= 28{\cdot}314$$

$$2{\cdot}8314 \times 100 = \frac{28314}{10000} \times 100$$

$$= \frac{28314}{100}$$

$$= \frac{20000}{100} + \frac{8000}{100} + \frac{300}{100} + \frac{10}{100} + \frac{4}{100}$$

$$= 200 + 80 + 3 + \frac{1}{10} + \frac{4}{100}$$

$$= 283{\cdot}14$$

$$\begin{aligned} 2{\cdot}8314 \times 1000 &= \frac{28314}{10000} \times 1000 \\ &= \frac{28314}{10} \\ &= \frac{20000}{10} + \frac{8000}{10} + \frac{300}{10} + \frac{10}{10} + \frac{4}{10} \\ &= 2000 + 800 + 30 + 1 + \frac{4}{10} \\ &= 2831{\cdot}4 \end{aligned}$$

$$\begin{aligned} \text{Thus we have, } 2{\cdot}8314 \times 10 &= 28{\cdot}314 \\ 2{\cdot}8314 \times 100 &= 283{\cdot}14 \\ 2{\cdot}8314 \times 1000 &= 2831{\cdot}4 \\ 2{\cdot}8314 \times 1000 &= 28314{\cdot}0 \end{aligned}$$

The pattern is in the shifting of the decimal point towards right in relation to the number of zeros in the multiplying power of 10.

When the number of zeros in the multiplying power of 10 is 1, the decimal point shifts or skips one place to the right.

When the number of zeros in the multiplying power of ten is 2, the decimal point shifts or skips two places to the right. When the number of zeros in the multiplying power of ten is 3, the decimal point shifts or skips three places to the right, and so on.

This pattern helps us to do multiplication by any power of 10 just by inspection.

Now let us take up division of a decimal fraction by a power of ten and study the pattern :

$$\begin{aligned} 4536{\cdot}7 \div 10 &= \frac{45367}{10} \times \frac{1}{10} = \frac{45367}{100} \\ &= \frac{40000}{100} + \frac{5000}{100} + \frac{300}{100} + \frac{60}{100} + \frac{7}{100} \end{aligned}$$

$$= 400+50+3+\frac{6}{10}+\frac{7}{100}$$

$$= 453{\cdot}67$$

$$4536{\cdot}7 \div 100 = \frac{45367}{10} \times \frac{1}{100} = \frac{45367}{1000}$$

$$= \frac{40000}{1000}+\frac{5000}{1000}+\frac{300}{1000}+\frac{60}{1000}+\frac{7}{1000}$$

$$= 40+5+\frac{3}{10}+\frac{6}{100}+\frac{7}{1000}$$

$$= 45{\cdot}367.$$

$$4536{\cdot}7 \div 1000 = \frac{45367}{10} \times \frac{1}{1000} = \frac{45367}{10000}$$

$$= \frac{40000}{10000}+\frac{5000}{10000}+\frac{300}{10000}+\frac{60}{10000}+\frac{7}{10000}$$

$$= 4+\frac{5}{10}+\frac{3}{100}+\frac{6}{1000}+\frac{7}{10000}$$

$$= 4{\cdot}5367.$$

Thus we have,

$4536{\cdot}7 \div 10 = 453{\cdot}67.$

$4536{\cdot}7 \div 100 = 45{\cdot}367.$

$4536{\cdot}7 \div 1000 = 4{\cdot}5367.$

By observing the pattern in the shifting of the decimal point in relation to the number of zeros in the dividing power of ten, we can make the following statements :

When the number of zeros in the dividing power of ten is 1, the decimal point shifts one place to the left.

When the number of zeros in the dividing power of ten is 2, the decimal point shifts two place to the left.

When the number of zeros in the dividing power of ten is 3, the decimal point shifts three places to the left.

This pattern helps up to do division by a power of ten just by inspection.

Multiplication of Decimal Fractions :

The method of multiplying any two decimal fractions is easily illustrated by the following examples :

Example : *1.* Find 12·84 × 5·6.

Solution : $$12{\cdot}84 \times 5{\cdot}6 = \frac{1284}{100} \times \frac{56}{10}$$
$$= \frac{1284 \times 56}{1000}$$
$$= \frac{71904}{1000}$$
$$= 71{\cdot}904.$$

Example : 2. Find 0·0731 × 0·452

Solution : $$0{\cdot}0731 \times 0{\cdot}452 = \frac{731}{10000} \times \frac{452}{1000}$$
$$= \frac{731 \times 452}{10000000}$$
$$= \frac{330412}{10000000}$$
$$= 0{\cdot}0330412.$$

These examples show that the multiplication of decimal fractions is like the multiplication of whole numbers, provided we know the place of the decimal point in the product. Let us review what we have seen above :

12·84 × 5.6 = 71·904
0·0731 × 0·452 = ·0330412
1·1101 × 0·001 = 0·0011101.

Discover the pattern in the number of decimal places on both sides.

It is seen that the total number of decimal places of the two decimal fractions multiplied is equal to the number of decimal places of the product.

Using this pattern, we can ignore the decimal point while multiplying two decimal fractions and fix it in the product after it is obtained.

Division of Decimal Fractions

Some simple examples of division are discussed here which illustrate well the division of one decimal fraction by another decimal fraction.

Example : *1* Find $2{\cdot}4 \div 2$.

Solution : $2{\cdot}4 \div 2 = \frac{2{\cdot}4}{2} = \frac{1}{2} \times 2{\cdot}4 = \frac{1}{2} \times \frac{24}{10} = \frac{24}{2} \times \frac{1}{10}$.

$$= \frac{12}{10} = 1{\cdot}2.$$

or $2{\cdot}4 \div 2 = \frac{2{\cdot}4}{2} = \frac{2{\cdot}4}{2} \times \frac{5}{5} = \frac{12{\cdot}0}{10} = 1{\cdot}2.$

Example : 2. Find $2{\cdot}4 \div 4$.

Solution : $2{\cdot}4 - 4 = \frac{2{\cdot}4}{4} = \frac{1}{4} \times 2{\cdot}4 = \frac{1}{4} \times \frac{24}{10} = \frac{24}{4} \times \frac{1}{10} = \frac{6}{10} = 0{\cdot}6.$

or $\frac{2{\cdot}4}{4} = \frac{2{\cdot}4 \times 5}{4 \times 5} = \frac{12}{20} = \frac{12}{2} \times \frac{1}{10} = \frac{6}{10} = 0{\cdot}6.$

Example : 3. Find $5{\cdot}5 \div 8$.

Solution : $5{\cdot}5 \div 8 = \frac{5{\cdot}5}{8} = \frac{1}{8} \times 5{\cdot}5 = \frac{1}{8} \times \frac{55}{10} = \frac{55}{8} \times \frac{1}{10}$

$$= \frac{6{\cdot}875}{1} \times \frac{1}{10} = 0{\cdot}6875.$$

or $$5{\cdot}5-8=\frac{5{\cdot}5}{8}=\frac{5{\cdot}5}{8}\times\frac{5}{5}=\frac{27{\cdot}5}{40}=\frac{27{\cdot}5}{4}\times\frac{1}{10}$$

$$=\frac{6{\cdot}875}{10}=0{\cdot}6875.$$

Example : *4*. Find 3·7 − 5.

Solution : $3{\cdot}7\div5=\frac{3{\cdot}7}{5}=\frac{3{\cdot}7}{5}\times\frac{2}{2}=\frac{7{\cdot}4}{10}=0{\cdot}74.$

Example : *5*. Find 2·6 − 0·2.

Solution : $2{\cdot}6\div0{\cdot}2=\frac{2{\cdot}6}{0{\cdot}2}=\frac{2{\cdot}6\times10}{0{\cdot}2\times10}=\frac{26}{2}=13.$

Example : *6*. Find 3·3 ÷ 0·04.

Solution : $3{\cdot}3\div{\cdot}04=\frac{3{\cdot}3}{0{\cdot}04}=\frac{3{\cdot}3\times100}{0{\cdot}04\times100}=\frac{330}{4}=82{\cdot}5.$

EXERCISE SET 3-15

Oral

1. Find
   (i) 4·186 × 100 (ii) 0·104 × 1000 (iii) 0·24 × 100
   (iv) 0·5 × 100 (v) 1·23 × 1000 (vi) 476·8 × 10.
2. Find
   (i) 4·186 − 100 (ii) 418·6 − 10 (iii) 2·4 ÷ 1000.
   (iv) 246 ÷ 10 (v) 138 ÷ 100 (vi) 138 ÷ 1000.
3. Using 0·16 × 2·41 = 0·3856, find :
   (i) 1·6 × 0·241 (ii) 16 × 0·0241 (iii) ·016 × 2·41
   (iv) 0·016 × 0·241 (v) 1·6 × 24·1.
4. By what smallest power of ten should the following be multiplied to get whole numbers :
   (i) 3·2 (ii) 0·28 (iii) 0·03 (iv) ·007 (v) 11·1 (vi) 11·01.

Written

5. Find the product :
   (i) 1·2 × 2·4 (ii) 1·2 × 24 (iii) 1·02 × 1·02
   (iv) 2·85 × 1·042 (v) 1·2 × 2·4 × 3·6 (vi) 1·2 × 3·12 × 0·8
   (vii) 0·072 × 0·009 (viii) ·01 × ·02 × ·003.

6. Find the square of · (i) 7·2 (ii) 2·51 (iii) ·025.

7. Find the cube of : (i) 0·2 (ii) 0·11 (iii) 1·5.

8. Rewrite each of the following by transforming it into division by a whole number :
   (i) 2·4 ÷ 0·8 (ii) 3·25 ÷ 0·05 (iii) 2·88 ÷ 1·2
   (iv) 10·24 ÷ 0·32 (v) 10·24 – 0·256 (vi) ·0072 ÷ 0·36.

9. Find the value of the following :
   (i) 4·6 – 2 (ii) 0·46 ÷ 2 (iii) 12·3 ÷ 3
   (iv) 1·23 ÷ 3 (v) 2·6 ÷ 5 (vi) 0·32 ÷ 5
   (vii) 0·32 ÷ 4 (viii) 1·64 ÷ 4 (ix) 5·15 – 5
   (x) 3·012 ÷ 3 (xi) 3·016 ÷ 8 (xii) 0·006 ÷ 6.

10. Find the value of the following :
    (i) 4·2 ÷ ·28 (ii) 2·88 ÷ 1·2 (iii) 8·64 – 3·6
    (iv) 10·24 ÷ 0·256 (v) 37·128 ÷ 3·64 (vi) 719·04 – 5·6

3-16 Some Applications of Decimal Fractions

Metric system of measures makes use of decimal fractions. This system can be displayed in a single table as given below. You know that we use metric system of measures in our country. Scietists all over the world also use this system and many countries have adopted it now.

Decimal System	*Metric System*
Thousandths	**Milli**
Hundredths	**Centi**
Tenths	**Deci**

Units	*Metre, Gram, Litre*
Ten	Deka
Hundreds	Hecto
Thousands	**Kilogram**

Denominations in bold types are in greater use in day to day transactions. More denominations are being added to meet the needs of Scientists in their study of microscopic things and vast space. Two important additions are the following :

one millionth of a unit is called a *micro* unit and a million of units a *mega* unit

Writing measures as decimal fractions of a unit, a sub-unit or a higher unit in metric system is easy.

For example,

1. 8 metres 5 decimetres 4 centimetres 4 millimetres
   = 8·543 metres
   = 85·43 decimetres
   = 854·3 centimetres
   = 8543 millimetres.

2. 6 kilograms 9 hectograms 2 dekagrams 5 grams
   = 6·925 kilograms
   = 69·25 hectograms
   = 692·5 dekagrams
   = 6925 grams

3 3461 litres = 346·1 dekalitres
= 34·61 hectolitres
= 3·461 kilolitres.

Estimation

You are asked to measure a given piece of wire, you are provided with a scale on which smallest subdivision in that of 1 cm. Suppose you find that the length is between 347 and 348 cms. What would be your measurement ? You shall use your intuition to find whether the length is nearer to 347 or 348. If it is nearer to 347, you shall say that the length is 347 cms. or 3.47 meters. Now you are given a scale on which smallest subdivision is that of 1 mm and you find the length exceeds 347 by a distance lying between 3 and 4 mm Then you shall again use your intuition for nearness and if you find that it is nearer 4 mm, then you shall say that the length is 347cm. 4 mm or 347. 4 cm. or 3·474 meters. When we write length of wire as 3·47. m, we shall say it is correct upto cms. and when we write 3·474 meters, we shall say that it is correct upto mms. Thus measurement of any kind is always on approximate measurement and depends upon the scales used. Further the estimate has to be made. *Finer the scales, better the approximation as well as estimate* The estimation plays an important role in calcultations.

Thus if a body weighs, 6·927 kilograms then.

wt. of body $= 6{\cdot}927$ correct upto $\frac{1}{1000}$ kilogram

$= 6{\cdot}93$ correct upto $\frac{1}{100}$ kilogram

$= 6{\cdot}9$ correct upto $\frac{1}{10}$ kilogram

$= 7$ correct upto kilograms.

EXERCISE SET 3-16

Oral

9. Read the following in metres :

(i) 3 metres 42 cm (ii) 14 metres 76 cm

(iii) 8 metres 6 decimetres 4 millimetres (iv) 9 metres 6 cm

(v) 7 metres 3 millimetres.

2. Read the following in centigrams :

   (i) 8 grams (ii) 3·47 grams (iii) 9.8 decigrams
   (iv) 73·4 dekagrams.

3. Answer the following in powers of 10 :

   (i) How many microlitres make a litre ?

   (ii) How many grams make a megagram ?

   (iii) How many kilometres make a megametre ?

4. Give the following approximations :

   (i) 9·487 metres correct to a centimetre.

   (ii) 5·349 kilograms correct to a hectogram.

   (iii) 19·47 litres correct to a litre.

5. Give the highest and lowest possible values of the original measurement for the corrected value given below taking one more place :

   (i) 6·1 litres (ii) 7·67 grams (iii) 5·0 metres
   (iv) 7·00 cm.

Written

6. How many micrometres make one megametre ? Give your answer in power of 10.

7. A box weighs 3·8 kilos. Find the weight of 45 boxes.

8. A vessel can hold 15·6 litres of milk. It contains 7·95 litres. How much more milk can be added ?

9. If 50 dolls weigh 3·2 kilos, find the weight of one doll. Give your answer in grams.

4

SENTENCES

4-1 Sentences

In everyday life, we often speak or write sentences such as :

(i) A dog is an animal;
(ii) New Delhi is the Capital of India;
(iii) A lion is a bird.

You see that the first two sentences are true, while the third is false. So you find that a sentence is either true or false. In Mathematics we write sentences such as .

(i) $5 + 3 = 8.$

(ii) $4 + 2 \neq \frac{25}{2}$,

(iii) $8 + 4 \neq \frac{24}{2}$.

(iv) every parallelogram is a rhombus

You know that the first two sentences are true while the third and fourth are false. In mathematics, we speak of such sentences as *mathematical sentences*. Just as in an ordinary sentence there is a verb, in a mathematical sentence there are symbols, such as, $=$ and $\neq$. You have already learnt that the sign $=$ means, 'is equal to' while $\neq$ means 'is not equal to'.

Like every statement, a mathematical sentence is either true or false but cannot be both.

4-2 Open Sentences

We now consider another type of mathematical sentences :

$$x + 5 = 7.$$

You cannot say readily whether this sentence is true or false. But you can do so when you know the value of x. We can give to x values that might make the sentence either a true one or a *false* one.

We can go on varying (changing) the value of x so that the sentence is either true or false. For this reason, x is called a variable. The variable x can vary over (or take values from) a specified *domain* or *replacement set* such as W or Q_+ . Sentences in which variables occur are called *open sentences.*

Although the sentence.

$$3x + 21 = 3(x + 7)$$

is true, for all values of x in the domain W or Q_+ , it is also an open sentence.

4-3 Solution Sets of Open Sentences

Consider the open sentence,

$$x + 5 = 7.$$

We take the set W of whole numbers as the domain (or the replacement set) ;

If x is 0, the sentence $0 + 5 = 7$ is false.

If x is 1, the sentence $1 + 5 = 7$ is false.

If x is 2, the sentence $2 + 5 = 7$ is true.

If x is 3, the sentence $3 + 5 = 7$ is false.

If x is 4, the sentence $4 + 7 = 7$ is false.

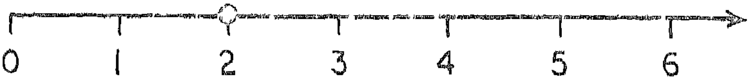
So you see that the set of whole numbers consists of two subsets, there is one subset, e.g., $\{ 2 \}$ for which the sentence is true while there is another subset $\{ 0, 1, 3, 4, ... \}$ for which the sentence is false. The subset $\{ 2 \}$ is called the solution set of the given open sentence.

4-4. Graphs of Solution Sets of Open Sentences

The representation of the solution set of an open sentence on the number ray is often very convenient. This is sometimes referred to as the *graph of the solution set of an open sentence.* The solution set { 2 } of the open sentence

$$x + 5 = 7,$$

is represented by the point 2 on the number-ray as shown below ·



EXERCISE SET 4-1

Oral

1. Examine whether the following sentences are true or false :

(i) $23 - 2 = 3$.　　(ii) $3 \times 10 = 30$.

(iii) $3 \times 10 = \frac{56}{2}$.　　(iv) $9 + 21 \neq \frac{60}{2}$.

2. The following are the open sentences with the domains assigned to each of them. Find the solution set for each of them :

(i) $x + 3 = 7$; { 5, 3, 1, 4, 0 }.

(ii) $3x \neq 0$; { 0, 2, 4, 8 }.

(iii) $0 \times x = 0$; W.

Written

3. Use the symbols =, ≠ in the following to get true

sentences :

(i) $5 + 3 \square 7 - 2$. (ii) $5 \times 3 \square 7 + 8$.

(iii) $5 \times 3 \square 5 \times 3$. (iv) $1 \times 0 \square 1 + 0$.

(v) $10 \div 0 \square 10 - 2$.

4 Find the solution set for each of the following sentences, $x \in W$.

(i) $x - 1 = 6$. (ii) $\frac{x}{5} \neq 6$.

(iii) $2x + 3 = 7$. (iv) $3x + 1 = x + 7$.

(v) $3x + 1 = 1$. (vi) $6x = 8$.

(vii) $\frac{2}{3}x = 0$. (viii) $10 = 2\frac{1}{2}x$.

5. Which of the following have empty solution sets ?

(i) $\frac{x}{2} = 3,\ x \in Q_+$. (ii) $\frac{x}{5} = 1,\ x \in Q_+$.

(iii) $x + 3 = 4,\ x \in W$. (iv) $x + 6 = 5,\ x \in W$.

(v) $2x = 7,\ x \in W$. (vi) $6x = 12,\ x \in W$.

4-5 Sentences with Other Connectives

We have talked about sentences with symbols $=$ and $\neq$.

We shall now consider sentences with other symbols. For example,

(i) $3 > 7$.

(ii) $3 < 1$.

(iii) $4 \not> 6$.

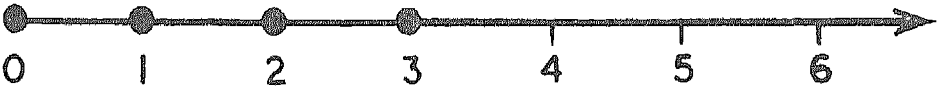
(iv) $5 \not< 0$.

These are sentences, with $>$, $<$, $\ngtr$, $\nless$, i. e., 'greater than', 'less than', 'not greater than', not less than'. Note that 'not greater than' means 'less than or equal to' denoted by $\leqslant$ and 'not less than' means 'greater than or equal to denoted by $\geqslant$. The first two sentences are false while the third and fourth are true.

An example of an open sentence with the symbol $<$ is the following ·

$$x + 3 < 7.$$

If we take the set of whole numbers W as the domain, then the solution set is clearly $\{ 0, 1, 2, 3, \}$. The graph for this is shown below :



EXERCISE SET 4-2

Oral

1. Say, if each of the following sentences is true or not :

(i) $8 < 11$. (ii) $4 > 0$.

(iii) $8 + 3 < 8 \times 3$. (iv) $1 \ngtr 1$.

(v) $2 \times 0 \nless 2 + 0$. (vi) $7 < 11$.

(vii) $1 + 1 \leqslant 2$.

2 The following are open sentences with the domain assigned to each of them. Find the solution set for each of them.

(i) $3x < 8$; $\{ 1, 2, 3, 4, \}$.

(ii) $4x - 3 > 10$; $\{ 0, 1, 2 \}$.

Written

3. Find the solution set in W of each of the following open sentences, and when it is not empty, draw the graphs.

(i) $x - 2 \nless 3$. (ii) $2x + 1 \ngtr 9$.

(iii) $x < 10$. (iv) $5x \ngtr 10$.

(v) $5x - 3 \leqslant 12$. (vi) $5 - 3x \geqslant 2$.

(vii) $15 \leqslant x - 1$.

4 See if each of the open sentences in the domains shown there, is true :

(i) $7x > 5$; $\{0, 1, 2, 3, 4\}$.

(ii) $3x > 8$; $\{0, 1, 2, 3, 4\}$.

(iii) $3x - 4 \ngtr 6$, $\{2, 4, 6\}$.

*5. Draw the graphs of the solution set of the following open sentences in W

(i) $x + 7 < 9$ (ii) $x - 2 < 6$ (iii) $3x > 12$.

(iv) $x + 2 = 2$. (v) $\frac{x}{2} < 5$

*6. Draw the graphs of the solution set of the following open sentences in the set of whole numbers W.

(i) $x + 7 > 9$. (ii) $x - 2 < 6$. (iii) $3x > 5$.

(iv) $x + 3 < 4\frac{1}{2}$. (v) $x > 7$.

4-6 Implications

Sometimes one statement leads to another. Suppose you are told that

Ram studies in the Town High School.

Does it mean that Ram's name is in the Town High School admission register ? Yes, it does. So the statement 'Ram studies in the Town High School' *implies* that 'Ram's name is in the Town High School admission register'. Is it also true to say that 'Ram's name is in the Town High School admission register' implies that 'Ram studies in the Town High School'. It need not be so as Ram might have left the school.

So when one statement implies another, we use the one way implication symbol → and it is read '*implies*'.

$$\left.\begin{array}{l}\text{Ram studies in the}\\ \text{Town High School}\end{array}\right\} \rightarrow \left\{\begin{array}{l}\text{Ram's name is in the}\\ \text{Town High School}\\ \text{admission register.}\end{array}\right.$$

When one statement implies a second statement and the second statement in turn implies the first statement, we use the two way, implication symbol ↔ and it is read as *implies and is implied by'* or simply '*is equivalent to*'.

Consider the following :

Statement 1 : A given whole number a is divisible by 4.

Statement 2 : A given whole number a is divisible by 2.

Does the statement 1 imply the statement 2 ?

Take the set of multiples of 4

$$\{ 4, 8, 12, 16, \ldots \}.$$

Each number in the set is divisible by 4 and also divisible by 2.

Now does the statement 2 imply the statement 1 ?

Take the set of multiples of 2

$$\{ 2, 4, 6, 8, 10, 12, \ldots\ldots \}.$$

There are many numbers such as 2, 6, 10, etc. which are divisible by 2 but not by 4.

So we cannot use the two way implication symbol for the statements 1 and 2.

We can only state :

$$\left.\begin{array}{l}\text{The number is}\\ \text{divisible by 4}\end{array}\right\} \rightarrow \text{The number is divisible by 2.}$$

Consider the following statements :

Statement 3 : $5 + 3 = 8$

Statement 4 : $5 = 8 - 3.$

Statement 3 implies statement 4 and statement 4 implies statement 3. There is therefore a two way implication and we can write.

$$5 + 3 = 8 \leftrightarrow 5 = 8 - 3.$$

This can also be written as :

$$5 + 3 = 8 \rightarrow 5 = 8 - 3 \rightarrow 5 + 3 = 8.$$

EXERCISE SET 4-3

Oral

1. Say if each of the following is true :
   - (i) 'Ram goes to school' implies 'it will not rain today'.
   - (ii) 'Gopal is Ramesh's son 'implies, Ramesh is Gopal's father'.
   - (iii) A given whole number *a* is divisible by 5 implies 'The given whole number *a* is divisible by 10'.
   - (iv) '$10 \times 4 = 40$' implies '$40 \div 4 = 10$'.

Written

2. Study the following statements and wherever there is implication, insert the symbol → or ↔ as the case may be :
   - (i) 'Ravi is Raghu's brother'; 'Raghu is Ravi's brother'.
   - (ii) 'John gets into Bombay Express'; 'John goes to Bombay'.

(iii) '$7 + 3 = 10$' ; '$6 \times 0 = 0$'

(iv) '$7 + 3 = 10$' ; '$10 - 3 = 7$'.

3. Study the following statements and wherever appropriate, change → into ↔ :

(i) Neela is Lata's daughter → Lata is Neela's mother.

(ii) $10^2 = 10 \times 10 \rightarrow 10^2 \div 10 = 10$.

(iii) $\left.\begin{array}{l}\text{The number } a \text{ is divisible}\\ \text{by 2.}\end{array}\right\} \rightarrow \left\{\begin{array}{l}\text{The number } a \text{ is}\\ \text{divisible by 5.}\end{array}\right.$

(iv) $\left.\begin{array}{l}\text{Two given numbers are}\\ \text{separately divisible}\\ \text{by 2.}\end{array}\right\} \rightarrow \left\{\begin{array}{l}\text{Sum of two given}\\ \text{numbers is divisi-}\\ \text{ble by 2.}\end{array}\right.$

4. Complete the following :

(i) $5+6=11 \rightarrow 5=\square$. (ii) $8\times3=24 \rightarrow 3=\square$.

(iii) $6-2=4 \rightarrow 6=\square$. (iv) $10 \div 2=5 \rightarrow 10=\square$.

(v) $1+0=1 \rightarrow 0=\square$.

4-7 Equations and Solution Sets :

We shall use the following properties of equality in transforming equations to simple equivalent equations.

Properties of Equality

(*1*) *Reflexive Property :*

A number is equal to it self. In symbols $a = a$.

Example : $5 = 5$.

(*2*) *Symmetric Property* :

Given two numbers, if the first is equal to the second, then the second is equal to the first.

In symbols if $a = b$, then $b = a$.

Example : $\frac{6}{8} = \frac{9}{12}$, $\frac{9}{12} = \frac{6}{8}$.

$6 = 8 - 2$, $8 - 2 = 6$.

(3) *Transitive Property* :

Given three numbers, if the first is equal to the second and the second is equal to the third, then the first is equal to the third.

In symbols, if $a = b$, and $b = c$, then $a = c$.

Examples : If $\frac{6}{8} = \frac{9}{12}$ and $\frac{9}{12} = \frac{15}{20}$.

then $$\frac{6}{8} = \frac{15}{20}.$$

If $2 + 3 = 4 + 1$ and $4 + 1 = 6 - 1$, then $2 + 3 = 6 - 1$.

Again $\frac{20}{2} = 5 \times 2$ and $5 \times 2 = 6 + 4$,

so $$\frac{20}{2} = 6 + 4.$$

(4) *Addition Property of Equality* :

An equality remains unchanged when the same number is added to both sides of it. In symbols,

if $a = b$, then $a + x = b + x$.

Example : $\frac{6}{4} = \frac{9}{6}$, then $\frac{6}{4} + \frac{1}{2} = \frac{9}{6} + \frac{1}{2}$.

An equality remains unchanged when the same number is subtracted from both sides of it, provided whole numbers or fractional numbers are got on both sides.

Example : $\frac{6}{4} = \frac{9}{6}$, then $\frac{6}{4} - 1 = \frac{9}{6} - 1$.

(5) *Multiplication Property of Equality* :

An equality remains unchanged when both of its sides are multiplied (or divided) by the same number x ($x \neq 0$ in the case of division).

In symbols :

if $a = b$, then $ax = bx$,

and $\frac{a}{x} = \frac{b}{x}$ $(x \neq 0)$.

Example : $4 + 1 = 6 - 1$,

then $3(4 + 1) = 3(6 - 1)$

and $\frac{4+1}{5} = \frac{6-1}{5}$.

Now let us solve a few equations by first transforming them into simple equivalent equations and then giving the solution set.

A member of the solution set of an equation is also called a root of the equation.

Example : Solve $x + 6 = 8$, where the replacement set is Q_+.

Solution : $x + 6 = 8$.

$x + 6 - 6 = 8 - 6$.

$x = 2$.

The solution set is { 2 }.

Verification : $2 + 6 = 8$.

Example : Solve $x - 2 = 3$, having the replacement set as Q_+.

Solution · $x - 2 = 3$.

$x - 2 + 2 = 3 + 2$

$x = 5$.

∴ The solution set is { 5 }.

Verification : $5 - 2 = 3$.

Example 3 : Solve $3x = 12$, given replacement set as Q_+.

Solution : $3x = 12$

$$\rightarrow \frac{3x}{3} = \frac{12}{3} \rightarrow x = 4$$

$\therefore$ The solution set is $\{ 4 \}$.

Example 4 : Solve $2x = 3$, having the replacement set as Q_+.

Solution : $2x = 3$

$$\rightarrow \frac{2x}{2} = \frac{3}{2} \rightarrow x = \frac{3}{2}.$$

$\therefore$ The solution set is $\left\{\frac{3}{2}\right\}$.

Verification : $2 \times \frac{3}{2} = 3$.

Note :—If the replacement set for $2x = 3$ be W, then the solution set is empty, as $\frac{3}{2} \notin W$.

Example 5 : Solve $\frac{x}{4} = 6$, having the replacement set as Q_+

$$\frac{x}{4} \times 4 = 6 \times 4.$$

$$x = 24.$$

$\therefore$ The solution set is $\{ 24 \}$

Verification : $\frac{24}{4} = 6$.

Example 6 : Solve $x \times 0 = 0$.

Solution : Every fractional number gives a true statement. So the solution set is Q_+ itself.

Where the solution set of an equation is equal to the replacement set, the equation is called an identity.

EXERCISE SET 4-4

Oral

1. Say if $x = 2$ is a root of the following equations :

   (i) $4x = 8$. (ii) $2x = 0$. (iii) $\frac{x}{4} = \frac{1}{2}$.

   (iv) $x + 1 = 1$. (v) $2x = \frac{1}{2}$. (vi) $x - 1 = 1$.

   (vii) $x - 2 = 0$. (viii) $3 - x = 0$. (ix) $x + 2 = 0$.

2. Transform the following equations into equivalent simple equations and give the root of each, $x \in W$:

   (i) $x + 5 = 6$. (ii) $x + 7 = 7$. (iii) $x + 1 = 3$.

   (ivi) $x + 4 = 4$. (v) $x + 8 = 12$. (vi) $x - 1 = 2$.

   (vii) $x - 3 = 3$. (viii) $x - 3 = 1$. (ix) $x - 4 = 0$.

3. Transform the following equations into equivalent simple equations and give the root of each, $x \in Q_+$:

   (i) $2x = 10$. (ii) $3x = 9$. (iii) $5x = 5$.

   (iv) $6x = 0$. (v) $2x = 1$. (vi) $3x = 5$.

   (vii) $4x = 2$. (viii) $6x = 9$. (ix) $9x = 3$.

   (x) $\frac{x}{4} = 1$ (xi) $\frac{x}{2} = 1$. (xii) $\frac{x}{3} = 3$.

Written

4. Translate each of the following sentences into equations and find the solution set in Q_+ :

   (i) a number increased by 3 is equal to 10.
   (ii) 8 subtracted from a number is equal to 7.
   (iii) 5 times a number is equal to 15.

(iv) 2 times a number is equal to 11.

(v) a number divided by 3 gives the quotient 12.

(vi) a number divided by 2 is equal to $\frac{1}{2}$.

5. Frame equations for the following and find the answer :

(i) If Raghav grows 10 cm more in height, he will be 180 cm tall. What is his height now ?

(ii) Shambhunath scored 32 marks and it was 8 marks less than the pass marks. What were the pass marks ?

(iii) 12 apples cost Rs. 9. Find the price of each.

6. Santa, Rama and Sheela are friends. Santa and Rama together weigh as much as Rama and Sheela weigh together. What can you conclude ?

7. There are four sticks say P, Q, R and S. The total length of P, Q, and R is the same as the total length of R and S. What can you conclude ?

8. Are the following statements true for numbers in Q_+ with the element 0 removed :

(i) $a > b \rightarrow ax > bx$

(ii) $a > b \rightarrow a + x > b + x$.

(iii) $a > b, b > c \rightarrow a > c$.

Point out those statements in this which are true for numbers in Q_+ also.

5

APPLICATIONS
SIMPLE CASES OF DIRECT VARIATION.

5-1 Introduction

You have so far studied the set of whole numbers and the set of fractional numbers. They are useful in solving many problems in our daily life such as buying and selling, lending and borrowing, etc. Let us start with a few examples.

Example : 1. One copy of a book costs Rs. 3. What is the cost of 20 copies of the book ?

It is Rs. 60.

What is the cost of 10 copies of the book ?

It is Rs. 30.

What is the cost of 15 copies of the book ?

It is Rs. 45.

We can give these in the form of a table as shown below :

Number of copies	1	20	10	15
Cost in Rs.	3	60	30	45

What do we find ? 20 copies are 20 times of one copy and Rs. 60 are 20 times of Rs. 3. 10 copies is half of 20 copies and Rs. 30 is half of Rs. 60.

When the number of copies is halved, the cost also gets halved.

15 copies is $1\frac{1}{2}$ times 10 copies and Rs. 45 is $1\frac{1}{2}$ times Rs. 30.

When the number of copies is increased to $1\frac{1}{2}$ times, the cost also gets increased to $1\frac{1}{2}$ times.

As the number of copies decreases, the cost also decreases *proportionately* and as the number of copies increases, the cost also increases *proportionately*.

Example: 2. Suresh walks at the rate of six kilometres per hour.

In 2 hours, he goes 12 km.

In 3 hours, he will go 18 km.

In 5 hours, he will go 30 km and so on.

We can, as before, give these results in the form of a table :

Duration in hours	1	2	3	5
Distance gone in km.	6	12	18	30

The duration of 3 hours is $1\frac{1}{2}$ times the duration of 2 hours. Correspondingly the distance of 18 km is $1\frac{1}{2}$ times the distance of 12 km.

The duration of 5 hours is $2\frac{1}{2}$ times the duration of 2 hours, correspondingly, the distance of 30 km is $2\frac{1}{2}$ times the distance of 12 km.

The duration of 3 hours is 3/5 of the duration of 5 hours; correspondingly, the distance of 18 km is 3/5 of the distance of 30 km.

As the duration increases, Suresh covers more distance *proportionately* As the duration decreases, Suresh covers less distance *proportionately.*

When two quantities are so related that as one increases, the other increases proportionately and as one decreases, the other decreases proportionately, one is said to vary directly as the other.

In example 1, the cost of copies varies directly as the number of copies.

In example 2, the distance covered by Suresh varies directly as the duration.

In example 1, let us denote the number of copies by x and the cost by y, then the variation of y with x is shown by the table :

x	1	20	10	15
y	3	60	30	45

In each case you see $\frac{y}{x} = 3$ or $y = 3x$. 3 is called the constant of variation. It is the price-rate of the book in this example.

In example 2, we shall denote the distance travelled by Suresh as y and the duration taken as x

Expressing their association in the form of a table

x	1	2	3	5
y	6	12	18	30

In each case $\frac{y}{x} = 6$ or $y = 6x$. 6 is called the constant of variation. It is the speed of walking of Suresh.

If two quantities x and y are so related that $\frac{y}{x}$ does not change, then y is said to vary directly as x.

It is written symbolically as $y \propto x$.

The constant of variation is usually denoted by the letter k.

Suppose in a varying situation, we know

$$\frac{y}{x} = 3.$$

Then we can write $y = 3\,x$, and in another situation

When $\frac{y}{x} = \frac{1}{2}$, we can write $y = \frac{1}{2}x$.

In general, if $\frac{y}{x} = k$, then $y = kx$.

Now consider the table of age and height of Khurshid ;

Age in years	10	20	30
Height in cm	125	160	170

As his age increases, his height increases. Can we say, his height varies directly as his age ? When his age becomes 2 times, his height does not become 2 times. When his age becomes 3 times, his height does not become 3 times.

So his height does not increase proportionately with his age.

Similarly, study the table of age and weight of a person or the table of height and weight of a person You will find that the weight of a person does not always vary directly as his age and the height of a person does not always vary directly as his weight.

EXERCISE SET 5-1

Oral

1. Study the following tables and say in each case if y varies directly as x :—

(i)

x	4	12	20	6	
y	7	21	35	10.5	

(ii)

x	12	6	9	18	3
y	18	9	13.5	27	4.5

(iii)

x	8	6	10	4	
y	12	10	14	8	

2. Say, if the following cases involve direct variation .

(i) It takes 10 minutes for 8 clothes to dry in the Sun. How long will it take for 16 clothes to dry in the Sun ?

(ii) Sen carries 3 litres of water in 2 similar water-bottles. How many such bottles does he need to carry 15 litres of water ?

Written

3. Find the constant of variation wherever it exists :

(i)

x	4	8	5	3
y	10	20	12.5	7.5

(ii)

x	7	5	8	10
y	21	25	32	30

4. Complete the following tables, given that $x \propto y$ in each

(i)

x	7	10	8	3	1
y	35				

(ii)

x	8	12	10	16	4
y		9			

5. Complete the table

(i) given $\frac{y}{x} = 4.$

x	6	10	5	4	
y					

(ii) given $\frac{y}{x} = \frac{2}{3}.$

x	6	3	9	4.5	
y					

6. If it takes 6 seconds for a wall-clock to strike 12, how long will it take to strike 6 ?

5-2 Interest and Principal

Suppose Deepak wants to construct a house. He needs money. So he goes to Neeraj to borrow money. Neeraj wants Deepak to pay Rs. 5 per year extra for every Rs. 100 given on loan.

For every Rs. 100 Deepak borrows, he will have to pay back at the end of a year, not only Rs. 100 but Rs. 5 more for using Neeraj's money for a year.

If he borrows Rs. 500, he will have to pay at the end of a year a sum of (Rs. 500 + Rs. 25).

If he borrows Rs. 5000, he will have to pay back

Rs. 5000 + Rs. 250.

In each case, he not only returns the loan but pays an additional sum for using another man's money.

This additional sum to be paid for using another man's money is called *interest.*

Thus if Deepak borrows money from Neeraj, Deepak is the *borrower* and Neeraj the *lender.* The sum borrowed or lent is called the *principal.* Extra money paid for using the principal for a certain period is the *interest.*

In the above example, when the principal is Rs. 100, the interest is Rs. 5 for a year.

When the principal is Rs. 500, the interest is Rs. 25 for a year.

When the principal is Rs. 5000, the interest is Rs. 250 for a year.

Neeraj charges Rs. 5 as interest per year on every Rs. 100/- he lends. This interest of Rs. 5 charged per year on every Rs. 100 is called the *rate of interest* As it is charged every year for every Rs. 100 it is more precisely called *rate percent per annum*

In our example, the rate of interest charged by Neeraj is 5 percent per annum, written briefly as *5% p. a.*

The rate of interest need not always be in the form of percent per annum. It may be *one paisa a month* on every rupee lent. In that case, the rate of interest is 1 *paisa per rupee per month.*

We may also keep our money in a bank. Bank will pay us interest. Money kept or invested in a bank is also called the principal.

It is usual to denote the Principal by P, the interest by I, the period by T and the rate of interest by R.

Taking another example, suppose the rate of interest is 4 percent per annum and the period is one year. We can then tabulate the values of P and I as follows :

P	100	200	300
I	4	8	12

When the rate of interest remains unchanged in a given period, I varies directly as P, that is, $\frac{I}{P}$ is constant.

Example: *1.* Find the interest on Rs. 500 when the rate of interest on Rs. 300 is Rs. 15.

Solution : We know interest varies directly with the principal.

$\frac{I}{P}$ is constant.

When $P = 300$, $I = 15$.

$$\therefore \frac{I}{P} = \frac{15}{300} = \frac{1}{20}.$$

$$\text{Now } I = \frac{1}{20} P.$$

When $P = 500$, we have to find I.

$$\therefore I = \frac{1}{20} \times 500 .$$

$$\leftrightarrow I = 25 .$$

Hence the interest on Rs. 500 is Rs. 25.

Example : 2. Find the rate per cent per annum of interest if year's interest on Rs. 400 is Rs. 16.

Solution :—We are given that when $P = 400$, $I = 16$.

We have to find I when $P = 100$.

The period is the same in both the cases.

Now we know $\frac{I}{P}$ is constant.

$$\therefore \frac{I}{P} = \frac{16}{400} = \frac{1}{25}.$$

$$\leftrightarrow I = \frac{1}{25} P.$$

When P is 100, $I = \frac{1}{25} \times \frac{100}{1} = 4.$

So the rate of interest is 4% p.a.

Example : 3. Find the interest on Rs. 650 for one year if the rate of interest is 3% p.a.

Solution :—If the rate of interest is 3% p.a., it means that when $P = 100$, $I = 3$ in one year,

when $P = 650$, we have to find I for one year.

The period is the same in both the cases.

We know $\frac{I}{P}$ is constant.

$$\therefore \frac{I}{P} = \frac{3}{100}.$$

$$\leftrightarrow I = \frac{3}{100} P.$$

When $P = 650$,

$$I = \frac{3}{100} \times 650$$

$$= \frac{1950}{100}$$

$$= 19{\cdot}50$$

Hence the interest on Rs. 650 is Rs. 19.50.

Example. 4 :—If the interest on Rs. 350 be Rs. 14, find the principal on which the interest will be Rs. 22.

Solution :—We are given that when $P = 350$, $I = 14$.

Since $\frac{I}{P}$ is constant,

$$\frac{I}{P} = \frac{14}{350} = \frac{1}{25}.$$

Now we have to find P, given $I = 22$.

We have $\frac{22}{P} = \frac{1}{25}$

$$\leftrightarrow P = 22 \times 25 = 550.$$

Hence the principal is Rs. 550

EXERCISE SET 5-2

Oral :

1. The interest on Rs. 200 for one year is Rs. 8 Find the interest for one year on

   (i) Rs. 400. (ii) Rs. 300. (iii) 250.

2. Find the rate percent per annum if a year's interest on Rs. 800 is Rs 16.

3. Find the interest on Rs. 500, for one year, if the rate of interest is 4% p.a.

4. The interest on Rs. 700 is Rs. 21. Find the principal when the interest is (i) Rs. 42. (ii) Rs. 63.

5. Interest is 2 paise per rupee per month. Find the monthly interest on (i) Rs. 100. (ii) Rs. 150. (iii) Rs. 250.

Written

6. Find the interest on Rs. 1250 for one year if the interest per annum on Rs. 650 be Rs. 32.50.

7. If the interest on Rs. 1350 be Rs. 54, find the interest on Rs. 850 for the same period.

8. Find the interest on Rs. 350 for a year if the rate of interest be 3%.

9. Find the interest on Rs. 750 for a year if the rate of interest be $7\frac{1}{2}\%$ p.a.

10. If the interest on Rs 1100 for a year be Rs. 44, find the rate percent per annum.

11. If the interest for a year on Rs. 1200 be Rs. 54, find the rate percent per annum.

12. Find the principal which gives the interest of Rs. 125 a year, if the rate of interest be 5% p.a.

13. Find the principal which gives the interest of Rs. 198 a year, if the rate of interest be $5\frac{1}{2}\%$ p.a.

14. If the interest on Rs. 1400 be Rs. 91, find the principal to yield an interest of Rs. 117 for the same period.

15. Find the principal which will give an interest of Rs. 165 in a year when the interest on Rs. 600 is Rs. 45 for one year.

Fill in the blanks, in the following (period being the same)

16. When $P = 100$, $I = 4$, then $P = 650$, $I =$ ______

17. When $P = 400$, $I = 24$, then $P = 700$, $I =$ ______

18. When $P = 800$, $I = 40$, then $P =$ ______, $I = 5$.

19. When $P = 600$, $I = 42$, then $P =$ ______, $I = 91$.

20. When $P = 750$, $I = 37.50$, then $P = 550$, $I =$ ______

5-3 Interest and Time

We shall see how interest and period of time vary when the principal and the rate of interest remain constant.

Principal Rs. 100 Rate of inerest 5% p.a

T	1	2	3	
I	5	10	15	

$\frac{I}{T}$ remains the same 5. So $\frac{I}{T}$ is constant.

Let us take another example :

Principal Rs. 300. Rate of interest 6% p. a.

T	1	2	3	
I	18	36	54	

$\frac{I}{T}$ remains the same 18. So $\frac{I}{T}$ is constant.

So when the principal and the rate of interest remain the same, interest varies directly as period of time.

Example : *1*. If the interest on a certain principal for 3 years be Rs. 75, find the interest on the same principal for 5 years.

Solution : When $T = 3$, $I = 75$.

$$\therefore \frac{I}{T} = \frac{75}{3} = 25,$$

$$\leftrightarrow I = 25\,T.$$

Now $T = 5$.

$$\therefore I = 25 \times 5 = 125.$$

Hence the interest for 5 years is Rs. 125.

Example : 2. If the interest on a certain principal for 4 years be Rs. 96, find the period in which the same principal will yield the interest of Rs. 144.

Solution : When $T = 4$, $I = 96$

$$\therefore \frac{I}{T} = \frac{96}{4} = 24$$

$$\leftrightarrow I = 24\,T$$

Now $I = 144$, We have to find T.

We get the equation $144 = 24\,T$

$$\leftrightarrow T = \frac{144}{24} = 6.$$

Hence the period of time required is 6 years.

EXERCISE SET 5-3

Oral

1. If the interest on a certian sum for 5 years is Rs. 200, find the interest on the same sum for (i) 2 years. (ii) 3 years. (iii) 4 years.

2. If a principal yields in 3 years an interest of Rs. 60, in what period will it yield
(i) Rs. 120   (ii) Rs. 90   (iii) Rs. 240 ?

Written

3. The interest on a certain principal for 4 years is Rs.120. Find the interest on the same principal for 7 years.

4. A certain principal yields Rs. 250 as interest for 5 years. What will be the interest on the same principal for 7 years ?

5. If the interest on a certain principal be Rs. 240 for 4 years, for what period will the same principal yield interest of Rs. 300 ?

Fill in the blanks (principal and rate of interest remaining the same).

6. When $I = 400$, $T = 5$, then $I = 480$, $T =$ ————
7. When $I = 560$, $T = 7$, then $I = 480$, $T =$ ————
8. When $I = 480$, $T = 8$, then $T = 5$, $I =$ ————
9. When $I = 360$, $T = 9$, then $T = 6$, $I =$ ————
10. When $I = 1250$, $T = 10$, then $T = 15$, $I =$ ————

5-4 Interest and Rate

Let us see how interest and rate vary when the principal and the period remain the same.

Principal Rs. 200   Period 1 year

R	5%	6%	7%	
I	10	12	14	

$\frac{I}{R}$ in the first case is $\frac{10}{5\%} = \frac{10}{5} \times 100 = 200.$

$\frac{I}{R}$ in the second case is $\frac{12}{6\%} = \frac{12}{6} \times 100 = 200.$

$\frac{I}{R}$ in the third case is $\frac{14}{7\%} = \frac{14}{7} \times 100 = 200.$

That is $\frac{I}{R}$ remains the same.

Let us take another example.

Principal Rs. 300 Period 2 years

R	5%	6%	8%	
I	30	36	48	

$\frac{I}{R}$ in the first case is $\frac{30}{5\%} = \frac{30}{5} \times 100 = 600.$

$\frac{I}{R}$ in the second case is $\frac{36}{6\%} = \frac{36}{6} \times 100 = 600.$

$\frac{I}{R}$ in the third case is $\frac{48}{8\%} = \frac{48}{8} \times 100 = 600.$

That is, $\frac{I}{R}$ remains the same.

Example : *1.* When the rate of interest is 5% p.a , the interest on a certain principal for one year is Rs. 30. Find the rate of interest if the interest on the same principal is Rs. 36 for one year.

Solution : When $R = 5\%$, $I = 30$.

We know $\frac{I}{R}$ is constant.

So $\frac{I}{R} = \frac{30}{5\%} = \frac{30}{5} \times 100 = 600.$

$\leftrightarrow I = 600\, R$

We are given $I = 36$, we have to find R.

So we get the equation,

$36 = 600\, R.$

$\leftrightarrow R = \frac{36}{600} = \frac{6}{100}\,.$

Hence the rate of interest is 6% p.a.

Example : 2. Find the interest in one year on a certain principal at 4% p.a., if the interest on the same principal be Rs. 42 at 7% p.a.

Solution : We are given that $I = 42$, when $R = 7\%$. We know $\frac{I}{R}$ is constant.

Now $\frac{I}{R} = \frac{42}{7\%} = \frac{42 \times 100}{7} = 600.$

$\leftrightarrow I = 600\, R$

We are given $R = 4\%$. We have to find I

$\therefore I = 600 \times 4\%.$

$= 600 \times \frac{4}{100}$

$= 24$

$\therefore$ the interest $= 24.$

EXERCISE SET 5-4

Oral

1. The interest on a certain sum at 4% is Rs. 150. What is the interest on the same sum at (i) 2% (ii) 8% (iii) 6%, the period in the two cases being the same.

2. When the interest is Rs. 200, the rate of interest is 6% Find the rate of interest, when the interest is (i) Rs. 400 (ii) Rs. 600, (iii) Rs. 100. The principal and the period of time remain the same.

Written

3. The interest on a certain principal is Rs. 200 at 7% p.a. If the interest on the same principal for one year is to be Rs. 10, find the rate of interest.

4 If the interest on a certain principal at 6% p.a. be Rs. 120, find the interest on the same principal at 8% p. a, the two periods being the same.

5. If the interest on a certain principal at 5% p.a. is Rs. 150, find the interest on the same principal if the rate of interest is increased to 6% p.a., the periods being the same.

6. When the interest on a certain principal at 4% p.a. be Rs. 160, find the interest on the same principal at 1% less rate, for the same period.

7. The interest on a principal at $4\frac{1}{2}\%$ p.a. is Rs. 180, find at what rate the same principal will yield Rs. 200 as interest in the same period.

Fill in the blanks (P and T remaining the same).

8. When $R = 4\%$, $I = 320$, then $R = 5\%$, $I =$ ――――

9. When $I = 300$, $R = 5\%$, then $I = 270$, $R =$ -

10. When $R = 4\%$, $I = 360$, then $R = 3\%$, $I =$ - - - -

5·5 Quantity and Cost

Suppose you buy a certain number of note-books at a certain cost. If you want to buy half the number of note books, then you pay only half the cost, or if you want to buy thrice the number of note books, you pay thrice the cost.

Consider another example. Suppose you buy a certain quantity of butter at a certain cost. If you want to buy $\frac{1}{4}$ of the quantity, you pay only $\frac{1}{4}$ of the cost, or, if you buy twice the quantity, you pay twice the cost.

So cost varies directly as quantity and the constant of variation is the price rate.

Representing the quantity by x, and the cost by y, we can write $\frac{y}{x}$ = Constant. Representing the constant by k, we have

$$\frac{y}{x} = k$$

$$\leftrightarrow y = k\,x.$$

Example : If the price of 10 litres of spirit is Rs 40, what is the price of 35 litres of spirit ?

Solution : Price varies directly as quantity.

Constant of variation is $\frac{y}{x} = \frac{40}{10} = 4$

$$\leftrightarrow y = 4\,x.$$

We are given the quantity as 35 litres.

We are asked to find the price.

That is $x = 35$

$\therefore y = 4 \times 35 = 140.$

Therefore the price of 35 litres of spirit is Rs. 140.

EXERCISE SET 5-5

ral

1. If the cost of 5 bats be Rs. 25, find the cost of
   (i) 10 bats. (ii) 15 bats. (iii) 7 bats.
2. If the cost of 100 bricks be Rs. 8, find the cost of
   (i) 400 bricks. (ii) 1000 bricks. (iii) 750 bricks.
3. If for Rs. 8, one can get 1 ream of superior quality paper, how much paper will one get if he spends
   (i) Rs. 16 (ii) Rs. 2 (iii) Rs. 20.

Written

4. If the cost of 7 tins of coffee be Rs. 35, find the cost of 25 tins of coffee.
5. Rama purchased a dozen pencils for Rs. 2.52. Find the cost of 10 pencils.
6. Sunil paid Rs. 1.50 for 3 apples. If he wanted to buy a dozen apples, what would he have to pay ?
7. A bookseller purchased 15 copies of Modern Algebra for schools and paid Rs. 37.50. Find the cost of one dozen copies of the same book.
8. Sunita purchased 480 g. of wool and paid Rs. 24. Find the cost of 1 kg of the same wool.
9. The cost of sugar is Rs. 210 per quintal. Find the cost per kilogram.

10. The cost of ghee is Rs. 12.50 per kg. Find the cost of 400 g of the same.

11. Fill up the blanks in the following table where x represents the weight of wheat flour in kg and y the cost of x kg of flour in rupees :

x	2.5	7.5	—	—
y	3	—	4.50	3.60

5-6 Time and Work

Suppose a tailor could stitch 3 trousers a day, he would then stitch 6 trousers in 2 days, 9 trousers in 3 days and so on.

Again, suppose a typist can type 1 sheet in 10 minutes, then he can type 6 sheets in 60 minutes, 9 sheets in 90 minutes and so on.

So if w units of work can be done by a person in d units of time, we find that

$$\frac{w}{d} \text{ is contant,}$$

that is, $\frac{w}{d} = k$, where k is the constant of variation.

It is rate of work in time and work problems.

Example : If a man can colour wash 30 sq. m of the surface of a wall in one day, in how many days will he colour-wash 270 sq. m of the wall ?

Solution :

When $w=30$, $d = 1$

$$\therefore \ \frac{w}{d} = 30$$

$\leftrightarrow w = 30\,d.$

When $w = 270$, we have to find d.

We get the equation

$$\begin{aligned} 270 &= 30\,d \\ \leftrightarrow d &= \frac{270}{30} \\ &= 9. \end{aligned}$$

Hence the man will colour-wash the surface in 9 days.

EXERCISE SET 5-6

Oral

1. If a carpenter makes 4 book-shelves a day, how many book-shelves will he make in 5 days ?
2. If a man can white-wash 100 sq. m of surface of a wall in a day, in how many days will he white-wash 1000 sq.m. of the surface of the wall ?
3. A tailor can make 9 shirts in 2 days. In how many days will he make 27 shirts ?
4. A tap can deliver 14 litres of water in 3 minutes. How long will it take to fill a tub of capacity of 21 litres ?

Written

5. A painter can paint 16 planks in 3 hours. How much time will he take to paint 20 planks ?
6. A typist can type a manuscript of 80 pages in 15 hours. How long will he take to type a manuscript of 140 pages ?
7. A farmer can sow 1000 sq.m. of a field in one day. How many days will he require to sow 4500 sq.m. ?

8. A Painter can paint 3 doors in 6 hours In how many hours will he complete painting of 4 doors.

9. A printing machine can print 400 sheets in 20 minutes. How much time will it take to print 1100 such sheets ?

10. A police dog squad is fed daily on 6 kg of dog food. How long will 48 kg of dog food last ?

5-7 Work and Number of Men

Jagdish wants to get a particular type of house built and employs 10 men. Suppose the house is built in a year. Then we can say that 10 men can build that particular type of house in 1 year. If 20 such men were employed, they could build 2 such houses in 1 year. If there were 30 such men, 3 such houses could be built in one year.

So, if m is the number of men required to do w units of work in a given period, then

$$w = km,$$

where k is the constant of variation.

Example : If 20 men can weave 160 metres of cloth in one day, how many metres of cloth can be woven by 50 men in a day ?

Solution : We know that $w = km$, or $\frac{w}{m} = k$.

When $m = 20,\ w = 160$

$$\therefore\ k = \frac{160}{20} = 8.$$

We are given $m = 50$.

We have to find w.

$\therefore\ w = 8 \times 50 = 400$

Hence 50 men can weave 400 metres of cloth.

EXERCISE SET 5-7

Oral

1. 20 men can lay one km of a road in one month. Find the length of the road that can be laid by 320 such men in one month ?

2. 5 boys can make 20 garlands a day. How many garlands a day can be made by 10 such boys ?

3. 3 men can build 10 m long wall in a day. What length of such a wall can 12 such men build in a day ?

4. 3 taps can deliver in 5 minutes 20 litres of water. How many more such taps should be installed if 60 litres of water is needed in 5 minutes ?

Written

5. 5 men can type 100 sheets a day. How many sheets a day can 17 men type ?

6. 25 men can weave 200 metres of cloth in a day. What quantity of cloth can be woven by 30 men in a day ?

7. 3 tailors can stitch 12 shirts in a day. Now fill in the blanks of the following sentences :

   (i) 10 tailors can stitch——— shirts in a day.

   (ii) ———tailors can stitch 20 shirts in a day.

   (iii) 30 tailors can stitch———shirts in a day.

   (iv) ———tailors can stitch 140 shirts in a day.

   (v) 43 tailors can stitch———shirts in a day.

5-8 Time and Distance

A cyclist goes at a speed of 8 km per hour. This means the cyclist covers a distance of 8 km every hour. So we can show by means of a table the distance he will cover in different periods of time. Let t denote the period of time in hours and d the distance in km. Then

t	2	3	4	
d	16	24	32	

It is clear that $d = kt$, k being constant.

The constant here is speed.

Example : If a train runs at a uniform speed of 20 km per hour, what distance will it cover in 4 hours.

Solution : Since the speed is given, we know k. It is 20.

So $d = 20\ t$

We are given $t = 4$. We have to find d.

$\therefore\ d = 20 \times 4 = 80.$

The train will cover *80 km* in *4* hours

EXERCISE SET 5-8

Oral

1. An aeroplane flies at a uniform speed of 800 km per hour. What distance does it cover in

(i) 2 hours. (ii) $\frac{1}{2}$ hour. (iii) $2\frac{1}{2}$ hours.

2. If a man walks 5 km in 2 hours, how long will he take to walk (i) 10 km. (ii) 2.5 km. (iii) 7.5 km.

Written

3. An aeroplane flies at a speed of 850 km per hour. What distance will it cover in $3\frac{1}{2}$ hours ?

4. A train moves at a speed of 45 km per hour. What distance will it cover in 4 hours ?

5. A boat covers 12 km. in 3 hours. What distance will it cover in 5 hours ?

6. An athlete runs 100 m in 15 seconds, then fill in the blanks in the following table, where d is the distance run in metres in time t :

t	5	10	20			
d				130	150	200

△
△△
△△△
△△△△

ANSWERS

Exercise Set 1-1 [Pages 3–6]

6. Set of animals having horns : { Buffalo, Deer }
   Set of animals not having horns :
   { Elephant, Lion, Tiger, Bear }

7. (a) A set of triangles.
   (b) A set of squares.
   (c) A set of circles.

Exercise Set 1-2 [Pages 10–12]

7. (i) $t \in H$
   (ii) { Seeta, Susan, Nalini, Neela }
   (iii) { Rivers of India }
   (iv) Bombay $\notin$ { Capitals of Countries }
   (v) $M=N$
   (vi) $A=\{0, 2, 4, 6, \ldots\}$
   (vii) $\{0, 2, 4, 6, \ldots, 100\}$.

8. (i) $\{1, 2, 3, \ldots, 7\}$ (ii) $\{7, 8, 9\}$.

9. $B=\{x|x$ is a counting number divisible by $3\}$
   $C=\{x|x$ is a whole number divisible by $4\}$.

10. (a) $\in$ (b) $\in$ (c) $\in$ (d) $\notin$ (e) $\notin$ (f) $\notin$

11. (i) $\in$ (ii) $\notin$ (iii) $\in$ (iv) $\in$ (v) $\notin$
    (vi) $\notin$ (vii) $\notin$ (viii) $\in$ (ix) $\notin$

12. (i) a (ii) e (iii) There is no such element.

Exercise Set 1-3 [Pages 15-17]

4. (i) $A \subset M$
   (ii) { Teachers in a school } $\not\subset$ { Pupils in a school }
   (iii) $\{a, e\} \subset \{x|x$ is a vowel $\}$.

5. (i) Elements are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 and 24.
(ii) Elements are 4, 7, 10, 13, 16, 19, 22, 25, 28 and 31.
(iii) Elements are 1, 3, 9, 27, 81, 243 and 729.
(iv) Elements are 1, 9, 25, 49, 81, 121 and 169.

6. (i) $\subset$ (ii) $\not\subset$ (iii) $\subset$ (iv) $\not\subset$ (v) $\subset$
(vi) $\subset$

7. (i) $=$ (ii) $\neq$ (iii) $=$

8. (i) $L=\{$ Tara, Nargis $\}$
(ii) $M=\{$ Ahmed, Mohan, Tara $\}$
(iii) $G=\{$ Tara $\}$
(iv) $H=\varnothing$

9. (i) Null set (ii) Every member of the subset.

10. (i) $\{a, b\}$, $\{a\}$, $\{b\}$, $\varnothing$
(ii) $\{e, f, g, h\}$, $\{e, f, g\}$, $\{e, g, h\}$, $\{f, g, h\}$, $\{e, f, h\}$, $\{e, f\}$, $\{e, g\}$, $\{e, h\}$, $\{f, g\}$, $\{f, h\}$, $\{g, h\}$, $\{e\}$, $\{f\}$, $\{g\}$, $\{h\}$, $\varnothing$

Exercise Set 1-4 [Pages 22-25]

5. (i) $D\cap G$ (ii) $E\cup F$

6. (i) $\{2, 3, 4, 5, 7\}$ (ii) $\{1, 2, 3, 4\}$ (iii) $\{a, b, c\}$
(iv) $\{a, b, c\}$ (v) { Ram, Mohan, Sita, Nutan }
(vi) $\{5\}$

7. (i) $\{2, 3\}$ (ii) $\{a, c\}$ (iii) $\varnothing$ (iv) $\{a\}$
(v) $\{\quad\}$ (vi) $\varnothing$ (vii) { Ram, Mahesh }

8. (i) $\{l, t, u\}$ (ii) $\{7, 9\}$

9. (i) $\{0, 1, 3, 4, 5, 6, 7\}$ (ii) $\{a, b, c, d, e\}$

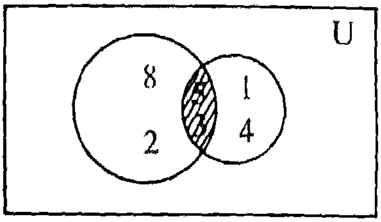
10. (i) $\{b, c\}$ (ii) $\{c, d\}$ (iii) $\varnothing$ (iv) $\{a, b, c, d\}$
(v) $\{b, c, d\}$ (vi) $\{a, b, c\}$ (vii) $\{a, b, c\}$
(viii) $\{a, b, c\}$

11. (i) { 1, 2, 3, 4, ..., 12 }   (ii) { 1, 3, 5, 7, 9, 11 }
(iii) { 1, 2, 3, 4, ..., 12 }   (iv) { 2, 3, 4, 5, 6, 7 }
(v) { 1, 2, 3, 4, ..., 12 }   (vi) { 3, 6, 9, 12 }

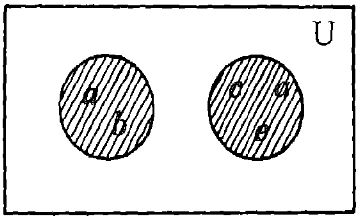
12. (i) ∩   (ii) ∪   (iii) ∪   (iv) ∩

13. (i) ⊂   (ii) ⊄   (iii) ∩

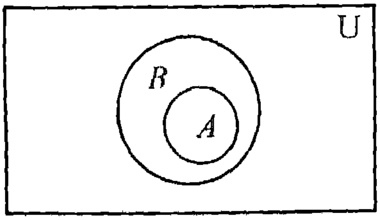
14. (i)



14. (ii)

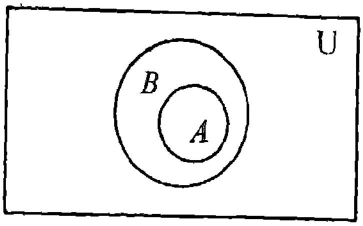


15. (i)



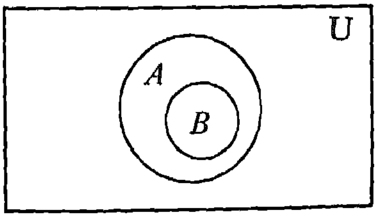
$A \cap B = A$
$A \cup B = B$

(ii)



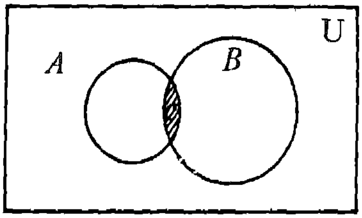
$A \subset B$

(iii)



$B \subset A$

(iv)



$C \subset A$

16. (i) Yes   (ii) not necessarily   (iii) $A = B$

17. (i) { 2 }   (ii) { 1, 2, 3, 4, 5, 6 }   (iii) { 2 }
(iv) { 1, 2, 3, 4, 5, 6 }

18. (i) A or B (ii) A or B

19. Union

20. $\varnothing$

21. (i) No (ii) No

Exercise Set 2 1 [Pages 29–31]

4. Girl.

5. A and D are equivalent sets.

6. Any two of the three diagrams, one containing birds, the other containing animals and the third containing names of hills.

Exercise Set 2 2 [Pages 35–36]

4. (i) $T \longleftrightarrow H$ (ii) $D \longleftrightarrow E \longleftrightarrow F$
(iii) $9 > 4$ (iv) $A \subset B \subset C$

5. (i) True (ii) False (iii) False (iv) True
(v) False (vi) True.

6. (i) Null
(ii) more (assuming that the sets K and L are finite)

7. $\{ a, b, c, d \}$ and $\varnothing$ are not equivalent sets.

$\{ a, b, c, \}$, $\{ a, b, d \}$, $\{ a, c, d \}$ and $\{ b, c, d \}$ are equivalent sets.

$\{ a, b \}$, $\{ a, c \}$, $\{ a, d \}$, $\{ b, c \}$, $\{ b, d \}$ and $\{ c, d \}$ are equivalent sets.

$\{ a \}$, $\{ b \}$, $\{ c \}$, and $\{ d \}$ are equivalent sets.

All the above sixteen sets are subsets of $\{ a, b, c, d \}$.

Exercise Set 2–3 [Pages 41–42]

5. (i) $n(\{ v, e, f, b, c \})$ (ii) $n(P) < n(G)$
(iii) 900803003.

6. (i) 481 tens and 4 ones. (ii) 48 Hundreds and 14 ones.

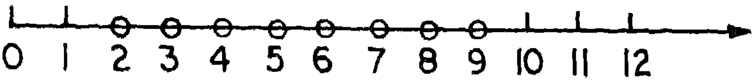
7 (i) $<$ (ii) $>$

8. (i) Ninety thousand (ii) Seven (iii) Equal.

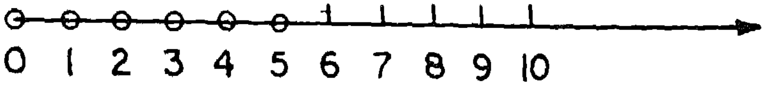
9. (i) The first (ii) No.

Exercise Set 2-4 [Pages 44-46]

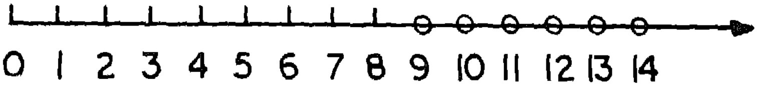
5.



6.



7.



8. $37 > 32$; $45 > 48$; $37 > 29$; $41 > 38$.

9. (i) { 0, 1, 2, ..., 10 } (ii) { 9, 10, 11, 12 }
(iii) { 7, 8, 9, ... } (iv) { 15 }.

Exercise Set 2-5 [Pages 49-50]

4. 155.

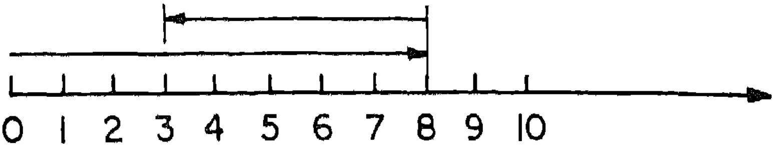
Exercise Set 2-6 [Pages 55-56]

3. (i) { 0, 1, 2, 3, 4, 5, 6, 7, 8) (ii) { 0 }
(iii) { 0, 1, 2, ... } (iv) { 1 }
(v) { 0 } (vi) { 6, 7, 8, ... } (vii) { 0,1,2,...,10 }.

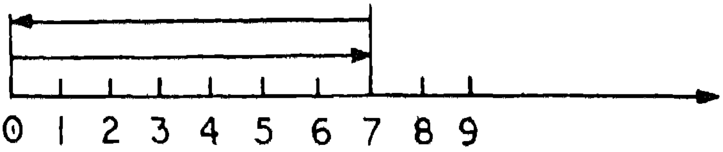
Exercise Set 2-8 [Pages 61-62]

5. $6 + 11 = 17$, $17 - 6 = 11$

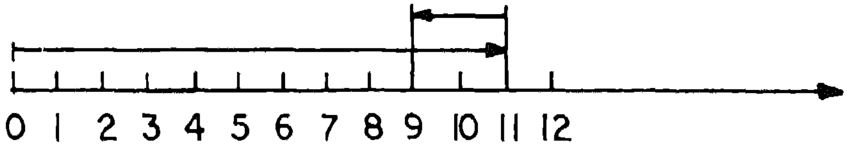
6. (i)



(ii)



(iii)



8. (i) 4 (ii) 7 (iii) 11 (iv) 6 (v) 13 (vi) 0.

9. (i) 0, 1 (ii) 0, 1, 2, 3 (iii) 0, 1, 2, 3, 4
(iv) 3, 4, 5, 6, 7 (v) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

10. (i) Yes (ii) No (iii) Yes (iv) No (v) Yes.

11. 0, 1, 2, 3, 4 and 5.

12. (i) 5 (ii) 4 (iii) 10 (iv) 8.

13. (i) True (ii) True (iii) True (iv) True
(v) True (vi) True.

Exercise Set 2-10 [Pages 65-66]

2. (i) Mangoes (ii) 4.

3. (i) How many centimetres is Vikram taller than his brother Kesav ?

(ii) How much more money does my father need to buy a house ?

4. 9270.

Exercise Set 2-11 [Pages 73-76]

5. (i)

× × × ×
× × × ×
× × × ×
× × × ×
× × × ×
× × × ×
× × × ×
× × × ×

(ii)

× × × × × × × ×
× × × × × × × ×
× × × × × × × ×
× × × × × × × ×

(iii)

×
×
×
×
×
×

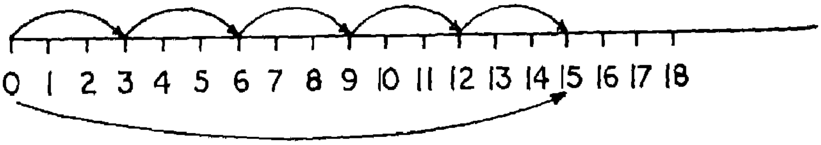
(iv)

× × × × × ×

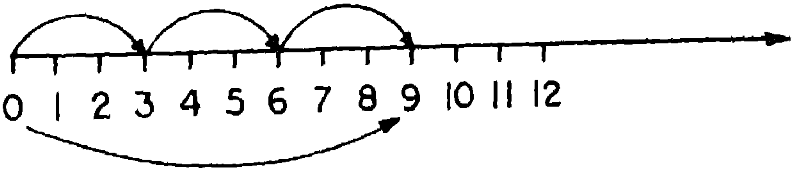
(v)

× × × × ×
× × × × ×
× × × × ×
× × × × ×
× × × × ×

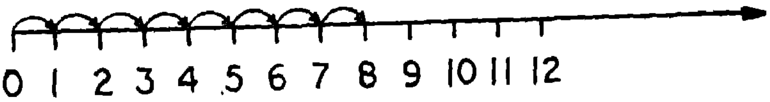
6. (i)



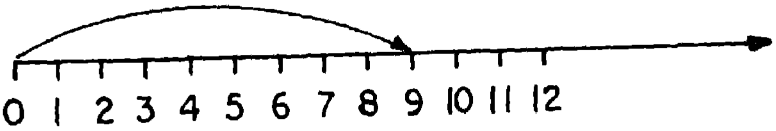
(ii)



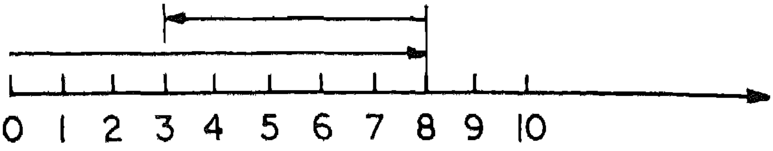
(iii)



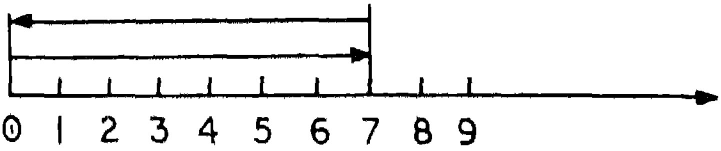
(iv)



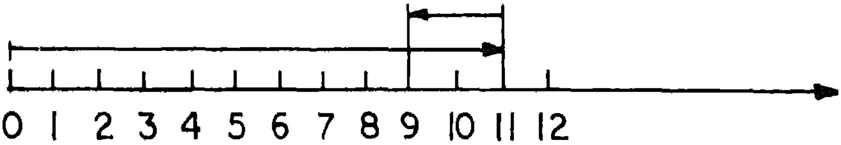
6. (i)



(ii)



(iii)



8. (i) 4 (ii) 7 (iii) 11 (iv) 6 (v) 13 (vi) 0.

9. (i) 0, 1 (ii) 0, 1, 2, 3 (iii) 0, 1, 2, 3, 4
(iv) 3, 4, 5, 6, 7 (v) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

10. (i) Yes (ii) No (iii) Yes (iv) No (v) Yes.

11. 0, 1, 2, 3, 4 and 5.

12. (i) 5 (ii) 4 (iii) 10 (iv) 8.

13. (i) True (ii) True (iii) True (iv) True
(v) True (vi) True.

Exercise Set 2-10 [Pages 65–66]

2. (i) Mangoes (ii) 4.

3. (i) How many centimetres is Vikram taller than his brother Kesav ?
   (ii) How much more money does my father need to buy a house ?

4. 9270.

Exercise Set 2-11 [Pages 73-76]

5. (i)

× × × ×
× × × ×
× × × ×
× × × ×
× × × ×
× × × ×
× × × ×
× × × ×

(ii)

× × × × × × × ×
× × × × × × × ×
× × × × × × × ×
× × × × × × × ×

(iii)

×
×
×
×
×
×

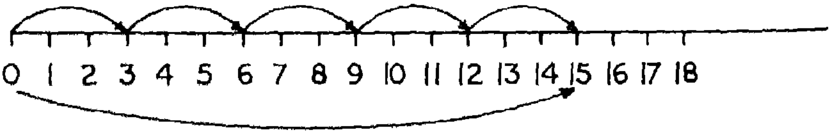
(iv)

× × × × × ×

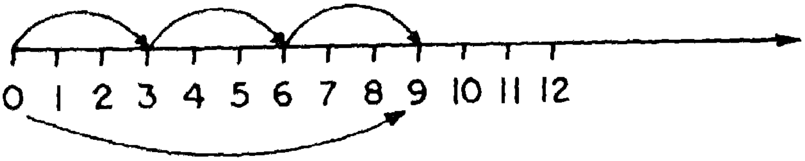
(v)

× × × × ×
× × × × ×
× × × × ×
× × × × ×
× × × × ×

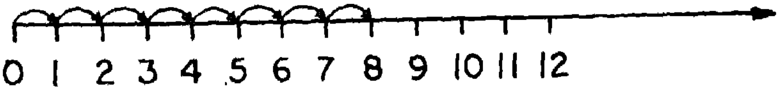
6. (i)



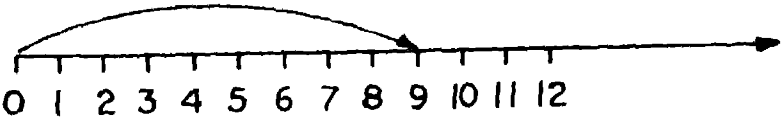
(ii)



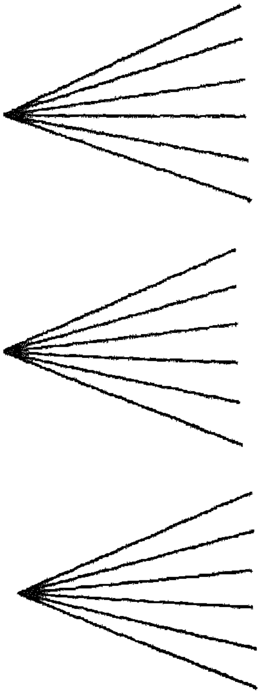
(iii)



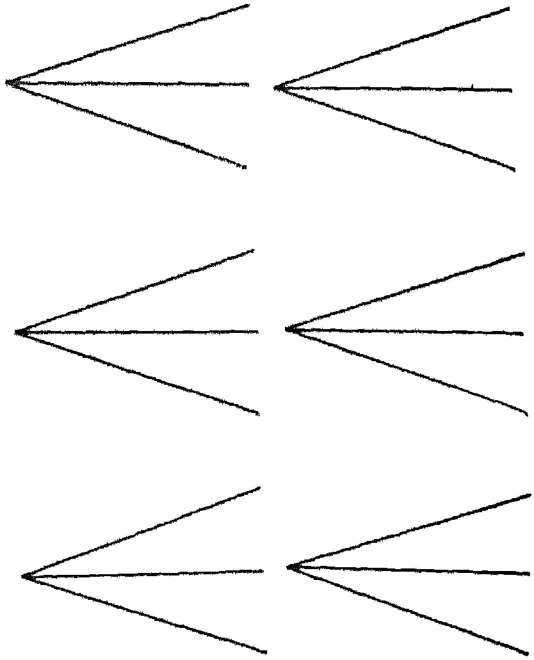
(iv)



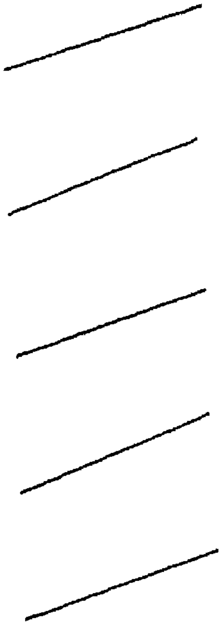
7. (i)



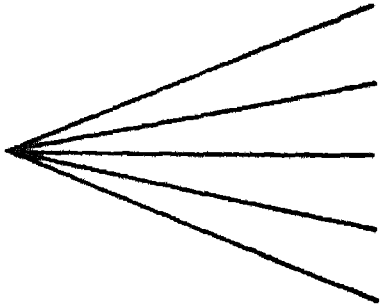
(ii)



(iii)



(iv)



8. (i) 30; 40; 5 × 10 = 50; 6 × 10 = 60
(ii) 600; 900; 12 × 100 = 1200; 15 × 100 = 1500
(iii) 5000; 7000; 9 × 1000 = 9000; 11 × 1000 = 11000.

9. (i) 1 × 12; 2 × 6; 3 × 4; 4 × 3; 6 × 2; 12 × 1
(ii) 1 × 15; 3 × 5; 5 × 3; 15 × 1.

10.

Solid	Dosa	Dosa	Idli	Idli	Puri	Puri
Drink	Coffee	Tea	Coffee	Tea	Coffee	Tea

Exercise Set 2–12 [Pages 82–84]

6. (i) True (ii) True (iii) True (iv) True.

7. (i) 98300 (ii) 649 (iii) 7894000.

8. (i) Yes (ii) Atleast one of the two numbers is zero.

9. (i) { 0, 1, 2, 3, 4, 5, 6, 7, 8 }
(ii) { 0, 1, 2, ..., 19 }
(iii) { 0, 1, 2, 3, ..., 8 }

Exercise Set 2–13 [Pages 86–88]

2. (i) 3 (5+6) (ii) 8 (9−1) (iv) $a(p+q) = ap+aq$
(v) $mn = nm$ (vi) (3+2) (3−2)

3. (i) 15 (ii) 27 (iii) 14 (iv) 17 (v) 12 (vi) 8

5. (i) 3×(4+1) (ii) (7×2)+6 (iii) (2×4) + (3×5)
(iv) 6×(5+2)×3 (v) 8×(5−3)

6. Rs. 6.00; Rs. 14.80; Rs. 8.40; Rs. 13.60; Total Rs. 42.80

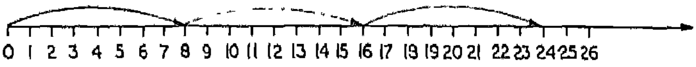
7. 3.60; 1.65; 12.05; 6.40; 23.00; Total 46.70.

Exercise Set 2–14 [Pages 93–95]

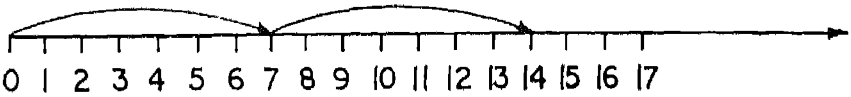
9. (i) $a = 21$, $b = 27$ (ii) $a = 52$, $b = 19$

(iii) $a = 133, \quad b = 0$ (iv) $a = 4, \quad b = 0$
(v) $a = 0, \quad b = 8$

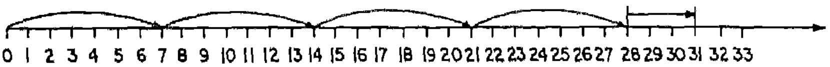
10. (i)



(ii)



(iii)



Exercise Set 2-15 [Pages 96-97]

(1) 157 (2) 24 (3) 1 (4) 30 (5) 3 (6) 92.

Exercise Set 2-16 [Pages 98-100]

6. (i) 8^5 (ii) 100^2 (iii) 1000^9 (iv) 6^{10}.

8. (i) 300581 (ii) 60408 (iii) 400000 (iv) 7002000.

9. (i) $10^2 \times 10^4 = 10^6$ (ii) $10^4 \times 10^3 = 10^7$
(iii) $10^7 \times 10^1 = 10^8$ (iv) $10^4 \times 10^4 = 10^8$.

10. (i) $10^5 \div 10^2 = 10^3$ (ii) $10^3 \div 10^1 = 10^2$
(iii) $10^4 \div 10^3 = 10^1$.

Exercise Set 2-17 [Pages 102-104]

10. (i) $1 \times 20, \quad 2 \times 10, \quad 4 \times 5, \quad 2 \times 2 \times 5$
(ii) $1 \times 48, \quad 2 \times 24, \quad 2 \times 3 \times 8, \quad 2 \times 3 \times 4 \times 2,$
$2 \times 3 \times 2 \times 2 \times 2, \quad 3 \times 16, \quad 3 \times 4 \times 4,$
$6 \times 8, \quad 4 \times 12, \quad 2 \times 2 \times 12, \quad 2 \times 2 \times 2 \times 6,$
$2 \times 4 \times 6.$
(iii) $1 \times 30, \quad 2 \times 15, \quad 3 \times 10, \quad 5 \times 6, \quad 2 \times 3 \times 5.$

11. (i) Any multiple of 30.

(ii) Any multiple of 60.
(iii) Any multiple of 200.

12. (i) 1, 2, 4 (ii) 1, 2, 4, 8.

13. 1, 4, 9, 16, 25, 36, 49, 64, 81.

14. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

15. The following digits occur in the unit place of square numbers :

0, 1, 4, 5, 6 and 9.

The following digits do not occur in the unit place of square numbers :

2, 3, 7 and 8.

Exercise Set 2-18 [Pages 106–107]

4. (i) $\because 0 = 2 \times 0$
i.e., 0 is a multiple of 2.
(ii) By adding 2 (iii) By adding 2.

5. (i)

$$1 + 3 + 5 + 7 + 9 + 11 = 36$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81$$

Property : Sum of consecutive odd numbers beginning with 1 is always a square of the number of terms added.

(ii)

$$5^2 - 4^2 = 9$$
$$6^2 - 5^2 = 11$$
$$7^2 - 6^2 = 13$$
$$8^2 - 7^2 = 15$$

Property : Difference of the squares of two consecutive whole numbers is always an odd number, which is equal to the sum of the two numbers.

6. Null set, i.e. $\varnothing$

7. No.

Exercise Set 2–19 [Pages 113–115]

5. $(2 \times 3 \times 5 \times 7 \times 11) + 1 = 2311$
$(2 \times 3 \times 5 \times 7 \times 11 \times 13) + 1 = 30031$
$(2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17) + 1 = 510511$
$(2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19) + 1$
$= 9699691.$

6. (i) $13 + 7$, $17 + 3$
(ii) $19 + 3$, $17 + 5$, $11 + 11$
(iii) $19 + 5$, $13 + 11$, $7 + 17$
(iv) $5 + 251$, $17 + 239$, $23 + 233$, $29 + 227$, $59 + 197$, $83 + 173$, $89 + 167$, $107 + 149$.
(v) $293 + 7$, $283 + 17$, $281 + 19$, $277 + 23$, $271 + 29$, $269 + 31$, $263 + 37$, $257 + 43$, $241 + 59$, $239 + 61$, $233 + 67$, $229 + 71$, $227 + 73$, $211 + 89$, $199 + 101$, $197 + 103$, $193 + 107$, $191 + 109$, $173 + 127$, $163 + 137$, $151 + 149$.

8. (i) $2 \times 2 \times 2 \times 2 \times 5$ (ii) $2 \times 2 \times 3 \times 3$
(iii) $3 \times 3 \times 7$ (iv) $2 \times 3 \times 5 \times 5$
(v) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$
(vi) 257 (vii) $7 \times 13 \times 19$ (viii) 11×101.

Exercise Set 2–20 [Pages 119–121]

6. (i) 3 (ii) $2 \times 3 \times 3$.

7. (i) $2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$
(ii) $2^3 \times 5 \times 7 \times 11$.

8. (i) 8 (ii) 21 (iii) 25.

9. (i) 3 (ii) 12 (iii) 10 (iv) 5 (v) 20
(vi) 4.

10. (i) 45 (ii) 60 (iii) 660 (iv) 24
(v) 7250 (vi) 7038.

11.

Number	Number	L.C.M.	H.C.F.	Product of Numbers	Product of L.C.M. and H.C.F.
12	20	60	4	240	240
84	60	420	12	5040	5040
24	40	120	8	960	960
40	56	280	8	2240	2240
24	56	168	8	1344	1344

Pattern : Product of numbers = Product of their L.C.M. and H.C.F.

12. 13, Number of boys' squad = 17
Number of girls' squad = 11.

13. 45 cm., Total number of bits is 7.

14. 14th September, 26th September.

15. Each number is 1.

16. 40.

Exercise Set 3-1 [Pages 125–128]

6. (i) $\frac{4}{5}, \frac{5}{5}$ (ii) $\frac{9}{10}, \frac{12}{10}, \frac{15}{10}$.

Exercise Set 3-2 [Pages 133–136]

4. (i) 16 (ii) 24 (iii) 8 (iv) 21.

5. (i) $\frac{15}{16}$ (ii) $\frac{8}{9}$ (iii) $\frac{4}{3}$ (iv) $\frac{31}{39}$ (v) $\frac{51}{110}$
(vi) $\frac{83}{164}$.

6. (i) Equivalent (ii) Equivalent.

7. (i) A_3 is associated with $\frac{3}{5}$

and A_5 is associated with $\frac{5}{5}$,

(ii) B_2 is associated with $\frac{2}{15}$,

B_5 is associated with $\frac{5}{15}$,

B_6 is associated with $\frac{6}{15}$,

B_{11} is associated with $\frac{11}{15}$

and B_{12} is associated with $\frac{12}{15}$.

8. (i) $\frac{1}{5}$ and $\frac{2}{10}$ (ii) $\frac{4}{6}$ and $\frac{2}{3}$.

9. (i) $\frac{1}{5}$ (ii) $\frac{69}{100}$ (iii) $\frac{3}{4}$.

Exercise Set 3-3 [Pages 141–143]

6. (i) $\frac{8}{12}$ (ii) $\frac{16}{20}$ (iii) $\frac{9}{10}$ (iv) $\frac{9}{5}$
(v) $\frac{11}{3}$ (vi) $\frac{4}{5}$.

7. (i) 4 bananas for 25 paise (ii) 7 oranges for 75 paise
(iii) 12 books for Rs. 36.

8. (i) If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$

(ii) If $\frac{a}{b} > \frac{c}{d}$, then $a \times d > b \times c$

(iii) If $\frac{a}{b} < \frac{c}{d}$, then $ad < bc$.

9. $\frac{8}{5} > 1$ and $\frac{7}{8} < 1 \rightarrow \frac{8}{5} > \frac{7}{8}$.

10. (i) $\frac{2}{5}$; $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$ (ii) $\frac{7}{9}$; $\frac{2}{3}$, $\frac{7}{9}$, $\frac{5}{6}$
(iii) $\frac{8}{5}$; $\frac{3}{2}$, $\frac{8}{5}$, $\frac{5}{3}$ (iv) $\frac{9}{9}$; $\frac{2}{5}$, $\frac{9}{9}$, $\frac{7}{4}$.

General Property : If $\frac{a}{b}$, $\frac{c}{d} \in Q_+$ and $\frac{a}{b} > \frac{c}{d}$,

then $\frac{a}{b} > \frac{a+c}{b+d} > \frac{c}{d}$.

Exercise Set 3–4 [Pages 148–151]

5. (i) $2+\frac{6}{7}$ (ii) $5+\frac{3}{5}$ (iii) $2+\frac{8}{11}$.

6. (i) $\frac{30}{7}$ (ii) $\frac{18}{5}$ (iii) $\frac{47}{8}$ (iv) $\frac{97}{11}$.

8. (i) $\frac{53}{60}$ (ii) $2\frac{1}{24}$ (iii) $1\frac{11}{48}$ (iv) $1\frac{3}{5}$
(v) $2\frac{65}{72}$ (vi) $3\frac{83}{120}$.

9. (i) Yes (ii) Yes (iii) Yes.

10. (a) (i) (7, 4) (ii) (3, 2) (iii) (8, 11)
(b) (i) 2 (ii) 6 (iii) 1 (iv) 1
(c) (8, 1)
(d) (i) (4, 1) (ii) (3, 5) (iii) (7, 6)
(iv) $(ad+bc, bd)$ (v) $(ac+ab, bc)$.
(e) No; it is true if $a=b$.

Exercise Set 3–5 [Pages 154–155]

3. (i) $7\frac{1}{20}$ (ii) $3\frac{8}{15}$ (iii) $6\frac{5}{8}$ (iv) 2.

4. (i) $8\frac{17}{22}$ (ii) $1\frac{47}{80}$ (iii) $\frac{49}{96}$.

5. $11\frac{534}{715}$.

6. $22\frac{79}{231}$.

Exercise Set 3–6 [Pages 158–161]

5. (i) $4\frac{5}{24}$ (ii) not possible in Q_+ (iii) $\frac{5}{48}$

(iv) $1\frac{1}{18}$ (v) $\frac{11}{12}$ (vi) $1\frac{14}{15}$.

6. (i) $\frac{3}{2}$; $\frac{3}{2}-\frac{2}{3}=\frac{5}{6}$; $\frac{3}{2}-\frac{5}{6}=\frac{2}{3}$

(ii) $\frac{19}{24}$; $\frac{19}{24}-\frac{5}{8}=\frac{1}{6}$; $\frac{19}{24}-\frac{1}{6}=\frac{5}{8}$

(iii) $\frac{77}{60}$; $\frac{77}{60}-\frac{7}{10}=\frac{7}{12}$; $\frac{77}{60}-\frac{7}{12}=\frac{7}{10}$

(iv) $\frac{11}{9}$; $\frac{11}{9}-1=\frac{2}{9}$; $\frac{11}{9}-\frac{2}{9}=1$

(v) $\frac{5}{18}$; $\frac{5}{18}-\frac{1}{6}=\frac{1}{9}$; $\frac{5}{18}-\frac{1}{9}=\frac{1}{6}$

(vi) $\frac{7}{2}$; $\frac{7}{2}-\frac{8}{3}=\frac{5}{6}$; $\frac{7}{2}-\frac{5}{6}=\frac{8}{3}$

(vii) $\frac{1}{4}$; $\frac{1}{4}-\frac{1}{4}=0$; $\frac{1}{4}-0=\frac{1}{4}$

(viii) $\frac{3}{11}$; $\frac{3}{11}-0=\frac{3}{11}$; $\frac{3}{11}-\frac{3}{11}=0$

(ix) 1; $1-\frac{2}{7}=\frac{5}{7}$; $1-\frac{5}{7}=\frac{2}{7}$.

7. (i) $\frac{1}{11}+\frac{3}{11}=\frac{4}{11}$; $\frac{4}{11}-\frac{1}{11}=\frac{3}{11}$

(ii) $\frac{8}{15}+\frac{1}{6}=\frac{7}{10}$; $\frac{7}{10}-\frac{8}{15}=\frac{1}{6}$

(iii) $3\frac{1}{4}+\frac{1}{4}=3\frac{1}{2}$; $3\frac{1}{2}-3\frac{1}{4}=\frac{1}{4}$

(iv) $\frac{7}{13}+\frac{6}{13}=1$; $1-\frac{7}{13}=\frac{6}{13}$.

8. (i) $\frac{1}{6}$ (ii) $\frac{7}{8}$ (iii) $4\frac{6}{11}$ (iv) 4 (v) $5\frac{7}{8}$ (vi) $\frac{3}{4}$.

9 (i) $\frac{3}{8}$ (ii) $\frac{1}{6}$ (iii) $3\frac{3}{5}$ (iv) 12 (v) $9\frac{3}{5}$ (vi) 18.

Exercise Set 3-7 [Pages 163–164]

2. (i) $\frac{4}{5}$ (ii) $\frac{2}{3}$ (iii) $3\frac{1}{3}$ (iv) $4\frac{1}{2}$ (v) $7\frac{1}{2}$ (vi) $1\frac{1}{5}$.

3 (i) $7\frac{1}{2}$ (ii) 15 (iii) $2\frac{1}{4}$ (iv) $9\frac{2}{7}$ (v) 54 (vi) 14

Exercise Set 3-8 [Pages 170-172]

4. (i) 14 (ii) 8 (iii) 24 (iv) 38 (v) 15 (vi) $1\frac{1}{3}$ (vii) $2\frac{1}{5}$ (viii) $3\frac{1}{4}$

5. (i) $15\frac{3}{8}$ (ii) $\frac{7}{9}$ (iii) $\frac{127}{220}$
(iv) $3\frac{1}{10}$ (Note : Read $11\frac{3}{5}$ in place of $1\frac{3}{5}$).

6. (i) $\frac{80}{231}$ (ii) 1 (iii) 1 (iv) 0 (v) 2 (vi) $2\frac{1}{8}$.

7. (i) $7\frac{1}{3}$ (ii) 20 (iii) 0 (iv) 9.

8. (i) $\frac{2}{3}$ (ii) $1\frac{1}{2}$ (iii) 23 (iv) 19.

9. Any positive rational number can be filled up in the frames.

General property : $\frac{a}{b} > \frac{c}{d} \to \frac{a}{b} \times x > \frac{c}{d} \times x$

if $x \in Q_+$.

10. (i) $\frac{11}{7}$ (ii) $24\frac{3}{5}$ (iii) $\frac{7}{37}$ (iv) $\frac{11}{86}$ (v) $\frac{7}{94}$
(vi) $4\frac{1}{5}$ (vii) $\left(\frac{7}{13}\right)^2$ (viii) $\left(\frac{5}{6}\right)^3$ (ix) $28\frac{4}{9}$
(x) $\frac{1}{484}$ (xi) not defined (xii) 1

Exercise Set 3–9 [Pages 173–174]

3. (i) $\frac{53}{216}$ (ii) 1 (iii) 1.

Exercise Set 3–10 [Pages 177–178]

3. (i) $\frac{4}{5}$ (ii) $\frac{5}{3}$ (iii) 2 (iv) $3\frac{1}{2}$ (v) $1\frac{2}{3}$
(vi) $\frac{10}{11}$.

Exercise Set 3–11 [Pages 179–180]

2. (i) 2 (ii) $\frac{4}{3}$ (iii) 3 (iv) $-\frac{28}{111}$ (v) $\frac{11}{20}$
(vi) $\frac{20}{11}$.

3. (i) $\frac{5}{8} \div \frac{3}{16} = \frac{10}{3}$ (ii) $1 \div 3 = \frac{1}{3}$.

Exercise Set 3–12 [Pages 182–184]

4. (i) 9 (ii) 25.

Exercise Set 3–13 [Pages 190–192]

8. (i) 761.44 (ii) 90.0095 (iii) $5 + \frac{16}{10} + \frac{3}{1000} = 6.603$
(iv) 8050.031.

9. (i) $2 \times 10 + 3 + \frac{7}{10} + \frac{1}{100} + \frac{3}{1000}$
(ii) $8 \times 100 + 1 \times 10 + 6 + \frac{3}{10} + \frac{3}{100}$
(iii) $8 \times 1 + \frac{1}{10} + \frac{6}{100} + \frac{3}{1000} + \frac{3}{10000}$
(iv) $\frac{7}{100}$ (v) $\frac{7}{100} + \frac{8}{1000} + \frac{4}{10000} + \frac{5}{100000}$
(vi) $\frac{4}{100} + \frac{6}{10000}$.

10. (i) 513·27 (ii) 35·086.

11. (i) $37\frac{1}{2}\%$ (ii) $62\frac{1}{2}\%$ (iii) $33\frac{1}{3}\%$
(iv) $66\frac{2}{3}\%$ (v) $6\frac{1}{4}\%$.

12 (i) $\frac{1}{15}$ (ii) $\frac{7}{8}$ (iii) $\frac{1}{6}$ (iv) $\frac{1}{11}$ (v) $\frac{1}{9}$.

13. (i) 2·05 (ii) 0 407 (iii) 0·047 (iv) 0·009
(v) 800·07 (vi) 3000 002 (vii) 407·06
(viii) 6060 0606.

14. (i) 5·23 (ii) 0 84 (iii) 0·248

15. (i) Second. (ii) Second alloy contains more gold and first alloy contains more copper.

16. (i) 5% (ii) Rs. 630.

Exercise Set 3–14 [Pages 194–195]

4 (i) 11·32 (ii) 7·27 (iii) 68·65 (iv) 682·45
(v) 0·0396 (vi) 150·05

6 (i) 37·29 (ii) 8·401 (iii) 0·0456 (iv) 238·002
(v) 5·0005 (vi) 10·9999 (vii) 8·202 (viii) 0·2106
(ix) 34 6986 (x) 0·54723.

7. (i) 35·87 (ii) 35·36 (iii) 0·96 (iv) 0·9696
(v) 0·9696 (vi) 0·9983 (vii) 2 196 (viii) 0·3654.

Exercise Set 3–15 [Pages 201–202]

5. (i) 2·88 (ii) 28·8 (iii) 1·0404 (iv) 2·9697
(v) 10·368 (vi) 2·9952 (vii) 0·000648 (viii) 0 0000006.

6. (i) 51 84 (ii) 6·3001 (iii) 0·000625.

7. (i) 0·008 (ii) 0·001331 (iii) 3·375

8. (i) 24 ÷ 8 (ii) 325 ÷ 5 (iii) 28·8 ÷ 12
(iv) 1024 ÷ 32 (v) 10240 ÷ 256 (vi) 0·72 ÷ 36.

9. (i) 2·3 (ii) 0·23 (iii) 4·1 (iv) 0·41
(v) 0·52 (vi) 0·064 (vii) 0·08 (viii) 0·41
(ix) 1·03 (x) 1·004 (xi) 0·377 (xii) 0·001.

10. (i) 15 (ii) 2·4 (iii) 2·4 (iv) 40 (v) 10·2
(vi) 128·4.

Exercise Set 3–16 [Pages 204–205]

6. 10^{12}.

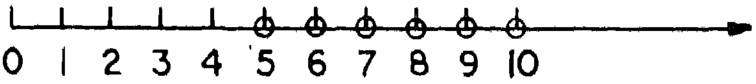
7. 171 kilos.

8. 7.65 litres.

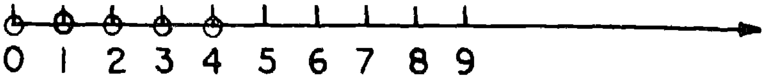
9. 64 grams.

Exercise Set 4–2 [Pages 210–211]

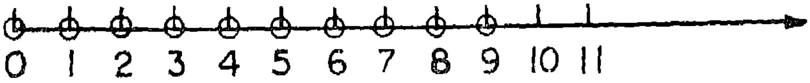
3. (i) { 5, 6, 7, 8, . . }



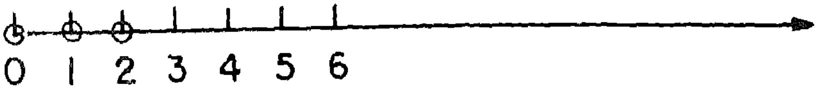
(ii) { 0, 1, 2, 3, 4 }



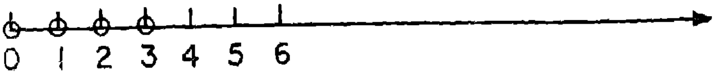
(iii) $\{0, 1, 2, \ldots, 9\}$



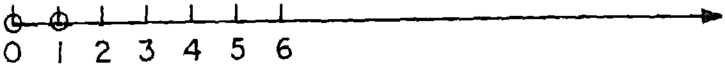
(iv) $\{0, 1, 2\}$



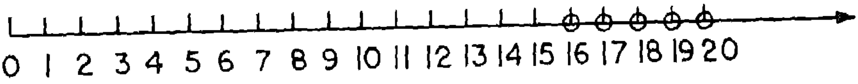
(v) $\{0, 1, 2, 3\}$



(vi) $\{0, 1\}$



(vii) $\{16, 17, 18, \ldots\}$



4. (i) No (ii) No (iii) No.

Exercise Set 4–3 [Pages 213–214]

2 (i) $\longleftrightarrow$ (ii) Does not imply (iii) Does not imply
(v) $\longleftrightarrow$.

3. (i) $\longleftrightarrow$ (ii) $\longleftrightarrow$ (iii) No change (iv) No change.

4. (i) $11 - 6$ (ii) $24 \div 8$ (iii) $4 + 2$
(iv) 5×2 (v) $1 - 1$.

Exercise Set 4–4 [Pages 218–219]

4. (i) $x + 3 = 10$; $\{7\}$ (ii) $x - 8 = 7$; $\{15\}$
(iii) $5x = 15$; $\{3\}$ (iv) $2x = 11$; $\{\frac{11}{2}\}$
(v) $x \div 3 = 12$; $\{36\}$ (vi) $x \div 2 = \frac{1}{2}$; $\{1\}$.

5. (i) $x + 10 = 180$; 170 cms (ii) $x - 8 = 32$; 40
(iii) $12x = 9$; Rs. $\frac{3}{4}$.

6. Weights of Santa and Sheela are the same.

7. Total length of P and Q is the same as that of S.

8. (i) True (ii) True (iii) True.
Statements (ii) and (iii) are true for numbers in Q_+.

Exercise Set 5-1 [Pages 224–226]

3. (i) $\frac{5}{2}$ (ii) does not exist.

4. Read '$y \propto x$' in each case in place of $\propto$ core x in each.

(i)

x	7	10	8	3	1
y	35	50	40	15	5

(ii)

x	8	12	10	16	4
y	6	9	7·5	12	3

5. (i)

x	6	10	5	4
y	24	40	20	16

(ii)

x	6	3	9	4·5
y	4	2	6	3

6. 3 seconds.

Exercise Set 5–2 [Pages 230–232]

6. Rs. 62·50
7. Rs. 34·00
8. Rs. 10·50
9. Rs. 56·25
10. 4% p.a.
11. $4\frac{1}{2}\%$ p.a.
12. Rs. 2500
13. Rs. 3600
14. Rs. 1800
15. Rs. 2200
16. 26
17. 42
18. 100
19. 1300
20. 27·50.

Exercise Set 5–3 [Pages 233–234]

3. Rs. 210
4. Rs. 350
5. 5 years
6. 6
7. 6
8. 300
9. 240
10. 1875.

Exercise Set 5–4 [Pages 237–238]

3. 2.31% p.a.
4. Rs. 160
5. Rs. 180
6. Rs. 120
7. 5% p.a.
8. 400
9. $4\frac{1}{2}\%$ p.a.
10. 270.

Exercise Set 5–5 [Pages 239–240]

4. Rs. 125
5. Rs. 2·10
6. Rs. 6·00
7. Rs. 30·00
8. Rs. 50·00
9. Rs. 2·10
10. Rs. 5·00
11.

x	2·5	7·5	3·75	3·0
y	3	9	4·50	3·60

Exercise Set 5–6 [Pages 241–242]

5. 3 hours 45 minutes.
6. 26 hours 15 minutes.
7. $4\frac{1}{2}$ days
8. 8 hours.
9. 55 minutes.
10. 8 days.

Exercise Set 5–7 [Page 243]

5. 340 sheets
6. 240 meters
7. (i) 40 (ii) 5 (iii) 120 (iv) 35 (v) 172.

Exercise Set 5–8 [Pages 244–245]

3. 2975 km
4. 180 km
5. 20 km
6.

t	5	10	20	19·5	22·5	30
d	$33\frac{1}{3}$	$66\frac{2}{3}$	$133\frac{1}{3}$	130	150	200